(8 pages)

Reg. No. : ....

Code No. : 41160 E **Sub. Code : JMMA 62/ JMMC 62**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Sixth Semester

Mathematics / Mathematics with CA — Main

COMPLEX ANALYSIS

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the best answer :

1. $\lim_{z \to -i} \frac{\bar{z} + z^2}{1 - \bar{z}} =$ ————.

(a) 1 (b) –1

(c) 0 (d) None

2. The function $f(z) = |z|^2$ is differentiable at ————.

(a) $z = 0$ only

(b) $z \neq 0$

(c) all points

(d) no points

3. The length l of a piecewise differential curve C given by equation $z = z(t)$, $a \leq t \leq b$ is ————.

(a) $\int_a^b |z(t)| dt$

(b) $\int_a^b |z'(t) dt|$

(c) $\int_a^b |z'(t)| dt$

(d) $\int_a^b |z(t) dt|$

4. If C denote the unit circle $|z|=1$, then $\int_C \frac{e^z}{z} dz =$ ________.

(a) 2π

(b) 0

(c) i

(d) $2\pi i$

5. The Taylor series expansion of $f(z)$ about the point zero is called ________.

(a) Zero-Taylor series

(b) Maclaurin's series

(c) Laurent's series

(d) None

6. $-z - z^2/2 - z^3/3 - \dots - z^n/n \quad \dots (|z|<1) =$ ________.

(a) $\cosh z$

(b) $\frac{1}{1-z}$

(c) $\log(1+z)$

(d) $\log(1-z)$

7. If $f(z) = \frac{ze^{iz}}{z^2+a^2}$, then $\text{Res}\,\{f(z); ia\} =$ ________.

(a) $e^a/2$

(b) $e^{ia}/2$

(c) $\frac{e^{-a}}{2}$

(d) $\frac{e^{-ia}}{2}$

8. If $f(z) = \frac{e^{iaz}}{(z^2+1)^2}$, then the poles of $f(z)$ are ________.

(a) $\pm 1, 0$

(b) $\pm 1, \pm i$

(c) $\pm i$

(d) $0, \pm i$

9. The transformation $w = \frac{1+z}{1-z}$ has fixed point ________.

(a) ± 1

(b) $\pm i$

(c) 0

(d) $1, i$

10. The four points z_1, z_2, z_3, z_4 are collinear, then $(z_1, z_2, z_3, z_4) =$ ________.

(a) 0

(b) complex

(c) real

(d) integer

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that $f(z) = \sqrt{r}\,(\cos \theta/2 + i \sin \theta/2)$, $r > 0$, $0 < \theta < 2\pi$ is differential and find $f'(z)$.

Or

(b) Find the constant a so that $u(x, y) = ax^2 - y^2 + xy$ is harmonic. Find an analytic function for which u is the real part.

12. (a) Evaluate the integral $\int_C (x^2 - iy^2)\,dz$ where C is the parabola $y = 2x^2$ from (1, 2) to (2, 8).

Or

(b) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\,dz$ where C is the circle $|z| = 3$.

13. (a) Expand $f(z) = \frac{z-1}{z+1}$ as a Taylor's series about the point $z = 1$. Determine the region of convergence in each case.

Or

(b) If $f(z) = \frac{z+4}{(z+3)(z-1)^2}$, find Laurent's series expansions in $0 < |z-1| < 4$.

14. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{13 + 5\sin\theta}$.

Or

(b) Prove that $\int_{-\infty}^{\infty} \frac{\sin x\,dx}{x^2 + 4x + 5} = \frac{-\pi \sin 2}{e}$.

15. (a) Show that by means of the transformation $w = 1/z$, the circle given by $|z - 3| = 5$ is mapped into the circle $\left|w + 3/16\right| = 5/16$.

Or

(b) Find the bilinear transformation which maps the points $z_1 = 2$, $z_2 = i$, $z_3 = -2$ onto $w_1 = 1$, $w_2 = i$, $w_3 = -1$ respectively.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Derive the CR-equations in polar coordinates.

Or

(b) Given the function $w = z^3$ where $w = y + iv$. Show that u and v satisfy CR-equations. Prove that the families of curves $u = c_1$ and $v = c_2$ (c_1 and c_2 – are constants) are orthogonal to each other.

17. (a) State and prove Cauchy's theorem.

Or

(b) State and prove Cauchy's integral formula.

18. (a) State and prove Taylor's theorem.

Or

(b) State and prove Laurent's theorem.

19. (a) Prove that $\int_0^\infty \frac{dx}{x^6+1} = \pi/3$.

Or

(b) Evaluate $\int_0^\infty \frac{dx}{1+x^4}$.

20. (a) Define a bilinear transformation. Prove that any bilinear transformation can be expressed as a product of translation, rotation, magnification or contraction and inversion.

Or

(b) Let f be an analytic function defined in a region D. Let $z_0 \in D$. Show that if $f'(z_0) \neq 0$, then f is conformal at z_0.

———————

(7 pages) **Reg. No. : ....**

Code No. : 41163 E **Sub. Code : JAMA 11/ SAMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

First/Third Semester

Mathematics

Allied — ALGEBRA AND DIFFERENTIAL EQUATIONS

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

SECTION A — $(10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. If $2+3i$ is a root of an equation with real coefficients, ten another root is ————.

 (a) $-2+3i$

 (b) $-2-3i$

 (c) $2-3i$

 (d) all these

2. If α, β, γ are the roots of the equation $x^3 + ax - b = 0$, then $\Sigma\alpha^3 =$ ________.

(a) a (b) b

(c) $-b$ (d) 0

3. If α, β, γ are the roots of the equation $x^3 - x^2 + x - 4 = 0$, then $-\alpha, -\beta, -\gamma$ are the roots of the equation ________.

(a) $-x^3 + x^2 - x + 4 = 0$

(b) $x^3 + x^2 + x + 4 = 0$

(c) $x^3 + x^2 + x - 4 = 0$

(d) $-x^3 + x^2 - x - 4 = 0$

4. The formula used in Newton's method to find an approximate solution is

(a) $\alpha_1 = \alpha - \dfrac{f(\alpha)}{f'(\alpha)}$ (b) $\alpha_1 = \alpha + \dfrac{f(\alpha)}{f'(\alpha)}$

(c) $\alpha_1 = \alpha - \dfrac{f'(\alpha)}{f(\alpha)}$ (d) $\alpha_1 = \alpha + \dfrac{f'(\alpha)}{f(\alpha)}$

5. The sum of eigen values of matrix $\begin{pmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{pmatrix}$ is

(a) 0 (b) 4

(c) 8 (d) –14

6. If $A = \begin{pmatrix} 3 & 0 & 0 \\ 3 & 4 & 0 \\ 3 & 6 & 1 \end{pmatrix}$, then the eigen values of A^3 are

(a) 3, 4, 1 (b) 9, 12, 3

(c) 27, 64, 1 (d) 9, 16, 1

7. If a and b are eliminated from $z = axy + b$, we get the partial differential equation

(a) $z = pxy + q$ (b) $py = qx$

(c) $px = qy$ (d) $p = q$

8. The auxillary equations of the equation $P_p + Q_q = R$ are

(a) $Pdx = Qdy = Rdz$ (b) $Pdx + Qdy = Rdz$

(c) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ (d) $\frac{p}{P} = \frac{q}{Q} = \frac{r}{R}$

9. $L(x) = 1$ ————————.

(a) 1 (b) s^2

(c) $\frac{1}{s^2}$ (d) $\frac{2}{s}$

10. $L^{-1}(F'(s)) =$ ————————.

(a) $xL^{-1}(F(s))$ (b) $-xL^{-1}(F(s))$

(c) $L^{-1}(F(s))$ (d) $L^{-1}(sF(s))$

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Solve the equation $x^5 - x^4 + 8x^2 - 9x - 15 = 0$ if $\sqrt{3}$ and $1-2i$ are two of its roots.

Or

(b) Show that the equation $x^3 + qx + r = 0$ will have one root twice another if $343r^2 + 36q^3 = 0$.

12. (a) Diminish the roots of the equation $x^3 + x^2 + x - 100 = 0$ by 4.

Or

(b) Find the Newton's method, the root of the equation $x^3 - 3x + 1 = 0$ which lies between 1 and 2.

13. (a) Find the characteristic equation of the matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

Or

(b) Find the sum and product of eigen values of the matrix $\begin{pmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{pmatrix}$.

14. (a) Solve $zp + x = 0$.

Or

(b) Find a partial differential equation by eliminating the arbitrary function f from $z = f\left(\frac{y}{x}\right)$.

15. (a) Find $L(xe^{-x}\cos x)$.

Or

(b) Find $L^{-1}\left(\frac{1}{s(s+1)(s+2)}\right)$

SECTION C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) If α, β, γ are the roots of the equation $x^3 + ax - b = 0$, find

(i) $\sum \frac{\alpha}{\beta\gamma}$ (ii) $\sum \frac{\alpha\beta}{\gamma}$

(iii) $\sum\left(\frac{\beta}{\gamma} + \frac{\gamma}{\beta}\right)$.

Or

(b) Solve : $2x^5 - 15x^4 + 37x^3 - 37x^2 + 15x - 2 = 0$.

17. (a) Find by correct to two places of decimals, the root of the equation $x^4 - 3x + 1 = 0$ that lies between 1 and 2 using Newton's method.

Or

(b) Find the positive root of $x^3 - x - 3 = 0$ correct to two places of decimals of Horner's method.

18. (a) If $A = \begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$, using Cayley Hamilton theorem find A^{-1}.

Or

(b) Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$.

19. (a) Form a partial differential equation : $z = f(2x + y) + g(3x - y)$.

Or

(b) Solve : $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$.

20. (a) Find : $L\left(\frac{1-\cos x}{x}\right)$.

Or

(b) Find :

(i) $L^{-1}\left(\frac{1}{(s+3)^2+25}\right)$

(ii) $L^{-1}\left(\frac{s}{(s+2)^2}\right)$.

(8 pages) **Reg. No. : ....**

Code No. : 41151 E **Sub. Code : JMMA 12/ JMMC 12/SMMA 12**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

First Semester

Mathematics / Mathematics with CA — Main

CLASSICAL ALGEBRA

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If the equation $2x^3 - 3x^2 + 2x - 3 = 0$ has one root 'i' then, its real root is ————————

   (a) $\frac{2}{3}$

   (b) $-\frac{2}{3}$

   (c) $\frac{3}{2}$

   (d) 1

2. The smallest degree of an equation with rational coefficients two of whose roots are $2+3i$ and $2-3i$ roots is ——————

(a) 2 (b) 4

(c) 6 (d) 3

3. The sum of the roots of the equation $x^4 - ax^3 + bx^2 - cx + d = 0$ is ——————

(a) $-\frac{b}{a}$ (b) $\frac{b}{a}$

(c) a (d) $-a$

4. A reciprocal equation $a_0 x^n + a_1 x^{n-1} + ... + a_n = 0$ is said to be of second type is ——————

(a) $a_{n-r} = a_{r-1}$

(b) $a_{n-r} = a_{r+1}$

(c) $a_{n-r} = a_r$

(d) $a_{n-r} = -a_r$

5. To remove the second term of $x^4 - 12x^3 + 48x^2 - 72x + 35 = 0$ the roots are to be diminished by ____________

(a) 1 (b) 2

(c) 3 (d) -1

6. If the roots of $x^3 - 8x^2 + 19x - 12 = 0$ are $1, 3, 4$ then the roots of $x^3 - 16x^2 + 76x - 96 = 0$ are ____________

(a) $1, 3, 4$ (b) $-1, -3, -4$

(c) $2, 6, 8$ (d) $1, 9, 16$

7. The negative roots of $f(x) = 0$ are ____________

(a) positive roots of $f(-x) = 0$

(b) positive roots of $f(-x) = -1$

(c) positive roots of $f(+x) = 0$

(d) negative roots of $f(-x) = 0$

8. If all the roots of $f(x) = 0$ are real then all the roots of $f'(x) = 0$ are ————

(a) imaginary

(b) real

(c) real or imaginary

(d) positive

9. Cardon's method deals with solving a ————

(a) quadratic equation

(b) cubic equation

(c) bi quadratic equation

(d) quintic equation

10. For the cubic equation $x^3 - 6x - 4 = 0$, the value of the discriminate is ————

(a) -16 (b) 14

(c) 52 (d) 48

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) If one root of the equation $2x^3 - 11x^2 + 38x - 39 = 0$ is $2 - 3i$. Solve the equation.

Or

(b) Solve the equation $4x^3 - 24x^2 + 23x + 18 = 0$, given that the roots are in arithmetic progression.

12. (a) If $\alpha + \beta + \gamma = 6$, $\alpha^2 + \beta^2 + \gamma^2 = 14$ and $\alpha^3 + \beta^3 + \gamma^3 = 36$ prove that, $\alpha^4 + \beta^4 + \gamma^4 = 98$.

Or

(b) Show that $4(x^2 - x + 1)^3 = 27x^2(x-1)^2$ is a standard reciprocal equation.

13. (a) Increase the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$ by 7.

Or

(b) Discuss the reality of the roots $x^4 + 4x^3 - 2x^2 - 12x + a = 0$ for all values of a.

14. (a) Find the multiple roots of $x^5 - x^4 + 2x^3 - 2x^2 + x - 1 = 0$ and hence solve.

Or

(b) Obtain by Newton's method the root of the equation $x^3 - 3x + 1 = 0$ which lies between 1 and 2.

15. (a) Solve $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$ using Ferrari's method.

Or

(b) Solve $2x^3 + 3x^2 + 3x + 1 = 0$ by Cardan's method.

SECTION C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) One root of the equation $2x^6 - 3x^5 + 5x^4 + 6x^3 - 27x + 81 = 0$ is $\sqrt{2} + i$. Find the remaining roots.

Or

(b) Show that the roots of the equation $px^3 + qx^2 + rx + s = 0$ are in G.P. iff $r^3 p = q^3 s$.

17. (a) State and prove Newton's theorem.

Or

(b) Solve: $6x^5 + 11x^4 - 33x^2 + 11x + 6 = 0$.

18. (a) State and prove Rolle's theorem.

Or

(b) Find the nature of the roots of $x^4 + 4x^3 - 20x^2 + 10 = 0$.

19. (a) Find the strum's functions for the polynomial $x^4 - 2x^3 - 3x^2 + 10x - 4$.

Or

(b) Find the positive root of the equation $x^3 - 2x^2 - 3x - 4 = 0$ correct to three places of decimals.

20. (a) Solve $x^3 - 3x^2 - 10x + 24 = 0$ using Cardan's method.

Or

(b) Solve $4x^4 + 8x^3 + 12x^2 + 4x + 5 = 0$ using Ferrari's method.

———————

(6 pages) **Reg. No. :** ....

Code No. : 40568 E **Sub. Code : SMMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

First Semester

Mathematics — Core

CALCULUS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. For any curve, the curvature is ————.

   (a) $\frac{ds}{d\chi}$ (b) $\frac{d\chi}{ds}$

   (c) $\frac{dy}{dx}$ (d) $\frac{ds}{dx}$

2. The radius of curvature of a circle of diameter d is

   (a) d (b) $\frac{2}{d}$

   (c) $\frac{d}{2}$ (d) $2d$

3. The $p-r$ equation of a parabola is ————.

   (a) $p^2 = ar$ (b) $p = ar$

   (c) $p = a^2r$ (d) $p^2 = ar^2$

4. The number of asymptotes of a general curve of n^{th} degree is

   (a) $n+1$ (b) $n-1$

   (c) $2n$ (d) n

5. If a double point to the curve $f(x, y) = 0$ is a conjugate point then ————.

   (a) $f_{xy}^2 < f_{xx} \cdot f_{yy}$ (b) $f_{xy}^2 > f_{xx} \cdot f_{yy}$

   (c) $f_{xy}^2 = f_{xx} \cdot f_{yy}$ (d) None

6. The curve $xy = c^2$ is symmetrical about ————.

   (a) x–axis (b) y–axis

   (c) $x = y$ (d) both the axes

7. The value of $\iint dy\,dx$ over the region $x \geq 0$; $y \geq 0$; $x + y \leq 1$ is

(a) $\frac{1}{3}$ (b) $-\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

8. Value of $\int_0^a \int_0^b \int_0^c dx\,dy\,dz$ is

(a) $a+b+c$ (b) abc

(c) $\frac{a+b+c}{2}$ (d) $a-b-c$

9. $\int_1^\infty \frac{dx}{x^2} =$ ————————.

(a) 1 (b) 0

(c) ∞ (d) 2

10. $\int_0^1 x^2(1-x)^3\,dx =$ ————————.

(a) $\frac{1}{5}$ (b) $\frac{1}{10}$

(c) 1 (d) $\frac{1}{60}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that the radius of curvature of the curve $y = c\cosh\left(x/c\right)$ at any point is $\frac{y^2}{c}$.

Or

(b) Find the formula for finding the radius of curvature of a curve expressed in polar form.

12. (a) Find the $(p-r)$ equation of the curve $r = a\sin\theta$.

Or

(b) Find all the asymptotes of the curves $x^3 - xy^2 + 6y^2 = 0$.

13. (a) Find the position and nature of the double points of the curve $x^2(x-y) + y^2 = 0$.

Or

(b) Trace the curve $y^2 = \frac{x^2(a+x)}{b-x}$.

14. (a) Evaluate $\iint xy\,dx\,dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$.

Or

(b) By changing into polar co-ordinates, evaluate $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} \left(x^2 + y^2\right) dx\,dy$.

15. (a) Evaluate $\int_{-1}^{1} \frac{dx}{x}$.

Or

(b) Evaluate $\int_{0}^{\infty} e^{-x^2} dx$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that the radius of curvature of $r^n = a^n \cos n\theta$ is $\frac{a^n r^{-n+1}}{n+1}$.

Or

(b) Show that in the parabola $y^2 = 4ax$ at the point 't', $\rho = -2a(1+t^2)^{3/2}$; $X = 2a + 3at^2$ and $Y = -2at^3$.

17. (a) Prove that the evolute of the cycloid $x = a(\theta - \sin\theta)$; $y = a(1-\cos\theta)$ is another cycloid.

Or

(b) Find the asymptoms of $x^3 + 2x^2y - 4xy^2 - 8y^3 - 4x + 8y = 1$.

18. (a) Prove that the curve $x^4 = y^2(x+y)$ has a double cusp of the first species at the origin.

Or

(b) Trace the curve $y = (x-1)(x-2)(x-3)$.

19. (a) Evaluate $\iiint xyz\, dx\, dy\, dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

Or

(b) Change the order of integration in $\int_0^a \int_y^a \frac{x\, dy\, dx}{x^2 + y^2}$ and evaluate it.

20. (a) Define $\Gamma(n)$ and prove that $\Gamma(n+1) = n!$ where n is a positive integer. Also test the convergence of $\Gamma(n)$.

Or

(b) State and prove relation between Beta and Gamma functions.

(7 pages) **Reg. No. :** ....

Code No. : 40581 E **Sub. Code : SSMA 4 A**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Fourth Semester

Mathematics

Skill Based Subject —TRIGONOMETRY, LAPLACE TRANSFORM AND FOURIER SERIES

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The coefficient of $\cos^n \theta$ in the expansion of $\cos n\theta$ is

   (a) 2^n (b) 2^{n-1}

   (c) 2^{n+1} (d) $2^n - 1$.

2. The coefficient of $\cos\theta$ in the expansion of $2^{n-1}\cos^n\theta$ is

   (a) ${}^nC_{\frac{n-1}{2}}$ (b) $\frac{1}{2}\,{}^nC_{\frac{n-1}{2}}$

   (c) ${}^nC_{n/2}$ (d) $\frac{1}{2}\,{}^nC_{n/2}$.

3. $\tan(ix) =$ ————.

   (a) $\tanh x$ (b) $i\tanh x$

   (c) $\frac{1}{i}\tanh x$ (d) $-i\tanh x$.

4. $\log 1 =$ ————.

   (a) 1 (b) 0

   (c) $i2n\pi$ (d) $n\pi$.

5. $L\left(e^{2t}\right) =$ ————.

   (a) $\frac{1}{s+2}$ (b) $\frac{1}{s}$

   (c) $\frac{1}{s-2}$ (d) 1.

6. $L^{-1}\left(\frac{a}{s^2+a^2}\right) =$ ————.

   (a) $\cos at$ (b) $\sin at$

   (c) $\cosh at$ (d) $\sinh at$.

7. $L(t) =$ ________.

(a) 0 (b) 1

(c) $\frac{1}{s}$ (d) $\frac{1}{s^2}$.

8. $L^{-1}\left(\frac{1}{s-3}\right) =$ ________.

(a) e^{3t} (b) e^{-3t}

(c) $t\,e^{3t}$ (d) $t\,e^{-3t}$.

9. Fourier coefficient of a_3 for $f(x) = x^3$ in $(-\pi, \pi)$ is

(a) 0 (b) $\frac{\pi^2}{3}$

(c) $\frac{2\pi^2}{3}$ (d) $\frac{2\pi^3}{3}$.

10. The Fourier coefficient a_0 for $f(x) = e^x$ in $(0, 2\pi)$ is

(a) 0 (b) $\frac{e^{2\pi} - 1}{2\pi}$

(c) $\frac{e^{2\pi} - 1}{\pi}$ (d) $\frac{e^{\pi} - e^{-\pi}}{\pi}$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Write $\cos 8\theta$ in terms of $\sin\theta$.

Or

(b) Expand $\sin^4\theta\cos^2\theta$ in a series of cosines of multiples of θ.

12. (a) Prove that $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$ if $\sin(A + iB) = x + iy$.

Or

(b) Separate into real and imaginary parts of $\tan^{-1}(x + iy)$.

13. (a) Find the value of $L\left(te^{-t}\cos t\right)$.

Or

(b) Find the value of $L\left(\frac{1 - \cos t}{t}\right)$.

14. (a) Find $L^{-1}\left(\frac{1}{(s+1)\left(s^2+2s+2\right)}\right)$.

Or

(b) Using Laplace transform, solve $y' - 5y = 0$, $y(0) = 2$.

15. (a) Find the Fourier series for

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 \le x < \pi. \end{cases}$$

Or

(b) Find the sine series of $f(x) = x$ in $(0, \pi)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9} = \frac{1}{8}$.

Or

(b) Give the expansion of $\cos^n\theta$ when n is a positive integer.

17. (a) Prove that $u = \log\tan(\pi/4 + \theta/2)$ if and only if $\cosh u = \sec\theta$.

Or

(b) Find $\sin\alpha + \sin(\alpha+\beta) + \cdots + \sin\left(\alpha + \overline{n-1}\,\beta\right)$.

18. (a) Find :

(i) $L\left[\frac{\sin at}{t}\right]$ and

(ii) $L\left(\frac{e^{-t}-1}{t}\right)$.

Or

(b) Find :

(i) $L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right]$.

(ii) $L^{-1}\left[\frac{s}{(s+2)^2}\right]$.

19. (a) Using Laplace transform, solve $y'' + 4y' + 13y = 2e^{-t}$, $y(0) = 0$, $y'(0) = -1$.

Or

(b) Using Laplace transform solve the equations $\frac{dx}{dt} + y = \sin t$; $\frac{dy}{dt} + x = \cos t$ given $x(0) = 2$, $y(0) = 0$.

20. (a) Show that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$ in the interval $-\pi \le x \le \pi$.

Or

(b) Find the Fourier cosine series for the function $f(x) = \pi - x$ in $(0, \pi)$.

(7 pages) Reg. No. : ....

Code No. : 40838 E **Sub. Code : GNMA 3A/ GNMC3A**

U.G. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Third Semester

Mathematics/ Mathematics with CA

Non-Major Elective — STATISTICAL METHODS

(For those who joined in July 2012–2015)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Let $-1 \le \gamma \le 1$. If we take $\gamma = 1$, then the correlation is
   (a) perfect and positive
   (b) perfect and negative
   (c) uncorrelated
   (d) none of these

2. If X and Y are independent variables, then $\gamma(X, Y) =$
   (a) 1 (b) –1
   (c) 0 (d) ∞

3. If $2x + 3y + 5 = 0$ be the regression line of y on x, then $b_{yx} =$
   (a) $-5/3$ (b) $-5/2$
   (c) $-2/3$ (d) $-3/2$

4. Arithmetic mean of the regression coefficient is ———— to the correlation coefficient.
   (a) $\ge$ (b) $\le$
   (c) $=$ (d) $\ne$

5. The relation between the difference operators is given by
   (a) $\nabla U_{x+h} = -\Delta U_x$ (b) $\Delta U_{x+h} = \Delta U_x$
   (c) $\nabla U_{x+h} = \Delta U_x$ (d) $\Delta U_{x+h} = -\Delta U_x$

6. $E =$
   (a) $1 + \Delta$ (b) $1 - \nabla$
   (c) $(1 + \Delta)^{-1}$ (d) $(1 - \nabla)^{-1}$

7. The values of x in U_x are not at equal intervals, we can use the formula ————

(a) Newton formula

(b) Gregory formula

(c) Lagrange formula

(d) None of these.

8. Given $N = 600$, $(A) = 300$, $(B) = 400$, $(AB) = 50$ then $(\alpha\beta) =$

(a) 50 (b) –50

(c) 40 (d) –40

9. If $Q = 1$, there is ———— association.

(a) perfect

(b) perfect positive

(c) perfect negative

(d) least

10. If $(A) \geq 0$ then A is ————

(a) inconsistent

(b) consistent

(c) frequency

(d) none

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Calculate the coefficient of correlation for the following data :

x :	160	161	162	163	164
y :	50	53	54	56	57

Or

(b) Derive Spearman's formula for rank correlation coefficient.

12. (a) Show that $\gamma = \pm\sqrt{b_{xy}\, b_{yx}}$.

Or

(b) If $x = 4y + 5$ and $y = kx + 4$ are the regression lines of x on y and y on x respectively, then show that $0 \leq k \leq 1/4$.

13. (a) If $U_0 = 1$, $U_1 = 5$, $U_2 = 8$, $U_3 = 3$, $U_4 = 7$, $U_5 = 0$ find $\Delta^5 U_0$.

Or

(b) Evaluate $\frac{\Delta^2 x^3}{Ex^2}$ taking $h = 1$.

14. (a) Find U_x from the given data using Newton's formula.

U_0	U_1	U_2	U_3
1	2	1	10

Or

(b) Using Lagrange's formula find a polynomial for the given data :

x :	0	1	3	4
U_x :	–12	0	6	12

15. (a) Given $(A) = 30$, $(B) = 25$, $(\alpha) = 30$, $(\alpha\beta) = 20$. Find

(i) N (ii) (β)

(iii) (AB) (iv) $(A\beta)$

(v) (αB).

Or

(b) Show that there is an error in the following data. 50% of people are wealthy and healthy, 35% are wealthy but not healthy, 20% are healthy but not wealthy.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that

$$\gamma_{xy} = \frac{n\Sigma x_i \Sigma y_i - \Sigma x_i \Sigma y_i}{\left[n\Sigma x_i^2 - (\Sigma x_i)^2\right]^{1/2}\left[n\Sigma y_i^2 - (\Sigma y_i)^2\right]^{1/2}}.$$

Or

(b) Ten students got the following percentage of marks in two subjects.

Economics :	78	65	36	98	25	75	82	90	62	39
Statistics :	84	53	51	91	60	68	62	86	58	47

Calculate the rank correlation coefficient.

17. (a) Find the equation of the regression line of y on x.

Or

(b) Find the equations of regression lines for the following data :

X :	25	28	30	32	35	36	38	39	42	45
Y :	20	26	29	30	25	18	26	35	35	46

(7 pages) **Reg. No. :** ....

Code No. : 40344 E **Sub. Code : JAMA 11/ SAMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

First/Third Semester

Mathematics — Allied

ALGEBRA AND DIFFERENTIAL EQUATIONS

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If α, β, γ are the roots of the equation $x^3 + 2x - 6 = 0$, then the value of $\alpha\beta\gamma$ is

(a) 0 (b) 2

(c) 6 (d) –6.

2. If $f(x) = 0$ is a reciprocal equation of first type and odd degree, then ———— is a factor of $f(x)$.

(a) $x+1$ (b) $x-1$

(c) x^2-1 (d) x^2+1.

3. The equation $x^4 - 3x^3 + 4x^2 - 2x + 1 = 0$ will be transformed by decreasing the roots by unity into the reciprocal equation

(a) $x^4 + x^3 + x^2 + x + 1 = 0$

(b) $x^4 - x^3 - x^2 - x + 1 = 0$

(c) $x^4 - x^3 + x^2 + x - 1 = 0$

(d) $x^4 - 4x^3 - 18x^2 - 3x + 2 = 0$.

4. If all the roots $f(x) = 0$ are real then all the roots $f'(x) = 0$ are

(a) imaginary (b) real and distinct

(c) real (d) real and imaginary.

5. The characteristic polynomial of $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ is

(a) $x^2 - 2x + 5 = 0$ (b) $x^2 + 2x + 5 = 0$

(c) $-x^2 - 2x + 5 = 0$ (d) $x^2 - 2x - 5 = 0$.

6. If the eigen values of a square matrix A are 1, 2, 3, then the eigen values of A^2 are

(a) 1, 4, 9 (b) 2, 4, 6

(c) −1, −4, −9 (d) 1, 1/2, 1/3.

7. The solution of $p^2 - 3p + 2 = 0$ is

(a) $(y - 2x + c_1)(y + x + c_2) = 0$

(b) $(y - 2x - c_1)(y - x - c_2) = 0$

(c) $(y - 3x - c_1)(y + 3x - c_2) = 0$

(d) $(y - 4x - c_1)(y + 4x - c_2) = 0$.

8. The partial differential equation obtained by eliminating 'f' from $z = f(x^2 + y^2)$ is

(a) $py^2 = qx^2$ (b) $px^2 = qy^2$

(c) $py = qx$ (d) $px = qy$.

9. $L[e^{-t}t^3] =$ ________

(a) $\frac{1!}{(s+1)^4}$ (b) $\frac{2!}{(s+1)^4}$

(c) $\frac{4!}{(s+1)^2}$ (d) $\frac{3!}{(s+1)^4}$.

10. $L^{-1}\left[\frac{1}{(s-4)^5}\right]=$ ——————

(a) $\frac{e^{4t}\,t^5}{25}$ (b) $\frac{e^{4t}\,t^4}{24}$

(c) $\frac{e^{-4t}\,t^4}{24}$ (d) $\frac{e^{-4t}\,t^5}{25}$.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve the equation $4x^3 - 24x^2 + 23x + 18 = 0$, given that the roots are in arithmetic progression.

Or

(b) Form the equation one of whose roots is $\sqrt{2}+\sqrt{3}$.

12. (a) Diminish the roots of $x^4+3x^3-2x^2-4x-3=0$ by 3.

Or

(b) Find the positive root of $x^3 - 6x + 4 = 0$ correct to two decimal places by Newton's *method*.

13. (a) Show that if λ is an eigen value of a non-singular matrix A, then $\frac{1}{\lambda}$ is an eigen value of A^{-1}.

Or

(b) Show that $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ satisfies the equation $A^2 - 2A - 5I = 0$ and hence find A^{-1}.

14. (a) Solve : $y = 2px + y^2p^3$.

Or

(b) Find the partial differential equation by eliminating the arbitrary function $xyz = \phi\left(x^2 + y^2 - z^2\right)$.

15. (a) Find $L\left[t \sin^2 t\right]$.

Or

(b) Find $L^{-1}\left[\log\left(\frac{s^2+9}{s^2+1}\right)\right]$.

PART C — $(5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve the equation

$$x^4 - 8x^3 + 7x^2 + 36x - 36 = 0,$$

given that two of its roots are equal in magnitude and opposite in sign.

Or

(b) Solve : $3x^6 + x^5 - 27x^4 + 27x^2 - x - 3 = 0$.

17. (a) Solve $x^4 + 20x^3 + 143x^2 + 430x + 462 = 0$ by removing its second term.

Or

(b) Find the positive root of $x^3 - x - 3 = 0$ correct to 2 decimal places by using Horner's method.

18. (a) Find the eigen values and the eigen vector of

$$A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix},$$

Or

(b) Verify Cayley-Hamilton theorem and hence find A^{-1} for the matrix $A = \begin{bmatrix} 3 & 3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$.

19. (a) Solve :

(i) $xp^2 - 2py + x = 0$.

(ii) $q - p = y - x$.

Or

(b) Solve : $(x + y)zp + (x - y)zq = x^2 + y^2$.

20. (a) Find :

(i) $L\left[\frac{e^{3t} - e^{-2t}}{t}\right]$.

(ii) $L^{-1}\left[\frac{2(s+1)}{(s^2 + 2s + 2)^2}\right]$.

Or

(b) Using Laplace transform solve $y'' + 6y' + 5y = e^{-2t}$ given that $y(0) = 0$ and $y'(0) = 1$.

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(6 pages) **Reg. No. :** ....

Code No. : 40335 E **Sub. Code : JMMA 31/ JMMC 31/SMMA 31**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Third Semester

Mathematics/ Mathematics with CA — Main

REAL ANALYSIS — I

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $x > y$ and $y > z$ then

   (a) $x = z$ (b) $z > x$

   (c) $x > z$ (d) $y > x$.

2. If a and b are real, then $|a+b| \geq$

   (a) $|a| + |b|$ (b) $|a| - |b|$

   (c) $||a| - |b||$ (d) $|b| - |a|$.

3. The range of the sequence $\left(1+(-1)^n\right)$ is

   (a) N (b) Z

   (c) $\{0, 1\}$ (d) $\{0, 2\}$.

4. $\lim_{n\to\infty} \frac{2n+1}{2n} =$

   (a) 0 (b) 1

   (c) 2 (d) –1.

5. $\lim_{n\to\infty} \frac{1}{n}\left(1+\frac{1}{2}+\cdots+\frac{1}{n}\right) =$

   (a) 0 (b) e

   (c) 1 (d) ∞.

6. If $(a_n) \to a$ and $(b_n) \to b$ then

   (a) $(a_n + b_n) \to a + b$ (b) $(a_n - b_n) \to a - b$

   (c) $(a_n / b_n) \to a/b$ (d) $(a_n) + (b_n) \to a + b$.

7. Let Σa_n be a series of positive terms. Then $n \to \infty$ is

   (a) convergent if $\lim_{n\to\infty} a_n^{1/n} > 1$

   (b) convergent of $\lim_{n\to\infty} a_n^{1/n} < 1$

   (c) divergent if $\lim_{n\to\infty} a_n^{1/n} < 1$

   (d) divergent if $\lim_{n\to\infty} a_n^{1/n} = 1$.

8. If $a_n = \frac{n!}{n^n}$ then $\lim_{n\to\infty} \frac{a_n}{a_{n+1}} =$

(a) e (b) 1

(c) 0 (d) $1/e$.

9. The series $\sum \frac{(-1)^{n+1} n}{5n+1}$

(a) converges (b) diverges

(c) oscillates (d) both (a) and (c).

10. For the geometric series Σx^n the radius of convergence R is

(a) 0 (b) 1

(c) ∞ (d) $1/n$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Write down the order axioms.

Or

(b) State and prove triangle inequality.

12. (a) Prove that any convergent sequence is bounded sequence.

Or

(b) If $(a_n) \to l$, $(b_n) \to l$ and $a_n \le c_n \le b_n$ for all n, then prove that $(c_n) \to l$.

13. (a) State and prove Cesaro's theorem.

Or

(b) Prove that every sequence (a_n) has a monotonic sequence.

14. (a) Discuss the convergence of the series $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \cdots$

Or

(b) State and prove Raabe's test.

15. (a) Show that the series

$$\frac{1}{2^3} - \frac{1}{3^3}(1+2) + \frac{1}{4^3}(1+2+3) - \frac{1}{5^3}(1+2+3+4) + \cdots$$

converges.

Or

(b) Find the radius of convergence, for the binomial series.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16\. (a) State and prove Additive property.

Or

(b) State and prove Cauchy-Schwarz inequality.

17\. (a) If $(a_n) \to a$ and $a_n \neq 0$ for all n and $a \neq 0$, then prove that $\left(\frac{1}{a_n}\right) \to \left(\frac{1}{a}\right)$.

Or

(b) Show that $\lim_{n\to\infty} \left(a^{1/n}\right) = 1$, where $a > 0$ is any real number.

18\. (a) Discuss the convergence of the geometric sequence (r^n).

Or

(b) Prove that

$$\frac{1}{n}\left[(n+1)(n+2)\cdots(n+n)\right]^{1/n} \to 4/e.$$

19\. (a) Prove that the harmonic series $\sum \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

Or

(b) State and prove Kummer's test.

20\. (a) Show that the series $\sum \frac{\sin n\theta}{n}$ converges for all values of θ and $\sum \frac{\cos n\theta}{n}$ converges if θ is not a multiple of 2π.

Or

(b) State and prove the Abel's theorem.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Answer should not exceed 600 words.

16. (a) State and prove the Binomial theorem.

Or

(b) If a_n is the n[th] Lucas number, then prove that $a_n < \left(\frac{7}{4}\right)^n$.

17. (a) State and prove the division algorithm.

Or

(b) Solve the linear Diophantine equation $172x + 20y = 1000$.

18. (a) State and prove the fundamental theorem of arithmetic.

Or

(b) If all the $n > 2$ terms of the arithmetic progression $p, p+d, p+2d, \ldots, p(n-1)d$ are prime numbers, then show that the common difference d is divisible by every prime $q < n$.

19. (a) State and prove the Chinese remainder theorem.

Or

(b) (i) Prove that 41 divides $2^{20} - 1$

(ii) Obtain the remainder when $1! + 2! + 3! + \ldots + 100!$ is divided by 12.

20. (a) State and prove Wilson's theorem.

Or

(b) Let p be an odd prime prove that quadratic congruence $x^2 + 1 \equiv 0 \,(md\ p)$ has a solution if and only if $p \equiv 1 \pmod 4$.

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Code No. : 41161 E **Sub. Code : JMMA 63/ JMMC 63**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Sixth Semester

Mathematics/Mathematics with CA — Main

NUMBER THEORY

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} \ldots + (-1)^n \binom{n}{n} =$

(a) n (b) 0

(c) $(-2)^n$ (d) 2^n

2. $0! =$

(a) 0 (b) ∞

(c) 1 (d) None

3. When we divide the square of any odd integer, the remainder is ————

(a) 1 (b) 3

(c) 5 (d) 7

4. $\gcd(-5,5) =$ ________

(a) 1 (b) –1

(c) –5 (d) 5

5. If P is a prime and p/ab, then ________

(a) p/a (b) p/b

(c) (a) or (b) (d) (a) and (b)

6. The number of prime numbers of the form $n^3 - 1$ is ________

(a) 1 (b) 0

(c) 7 (d) ∞

7. $-31 \equiv$ ________ $(\text{mod } 7)$

(a) 10 (b) 11

(c) –4 (d) –5

8. If $ca \equiv cb (\text{mod } n)$ and $\gcd(c,n)=1$, then ________

(a) $a \equiv b (\text{mod } n)$ (b) $a \equiv c (\text{mod } n)$

(c) $b \equiv c (\text{mod } n)$ (d) $a \equiv b\left(\text{mod } \frac{n}{c}\right)$

9. If p is a prime and a is any integer, then $a^p \equiv$ ________ $(\text{mod } p)$

(a) 1 (b) 0

(c) a (d) p

10. $12! + \equiv$ ________ $(\text{mod } 13)$

(a) –1 (b) 1

(c) 12 (d) 0

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Answer should not exceed 250 words.

11. (a) State and prove the Archimedean property.

Or

(b) If t_n is the n^{th} triangular number, prove that
$$t_1 + t_2 + \ldots + t_n = \frac{n(n+1)(n+2)}{6}, n \geq 1$$

12. (a) (i) If $\gcd(a,b)=1$, then show that $\left(\frac{a}{d}, \frac{b}{d}\right)=1$.

(ii) State and prove the Euclid's lemma.

Or

(b) Prove : $\gcd(a, b) \times \text{lcm}(a, b) = ab$.

13. (a) Show that $\sqrt{2}$ is irrational number.

Or

(b) Prove that there are an infinite number of primes of the form $4n+3$.

14. (a) Prove that $a \equiv b (\text{mod } n)$ if and only if a and b leave the same non negative remainder when divided by n.

Or

(b) Solve the system:
$7x + 3y \equiv 10 (\text{mod } 16)$, $2x + 5y \equiv 9 (\text{mod } 16)$.

15. (a) Show that the converse of Fermat's theorem is not true.

Or

(b) If n is an odd pseudo prime, then show that $M_n = 2^n - 1$ is also an odd pseudo prime.

(8 pages) **Reg. No. :** ....

Code No. : 40345 E **Sub. Code : JAMA 21/ SAMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Second/Fourth Semester

Mathematics — Allied

VECTOR CALCULUS AND FOURIER SERIES

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $\overline{A} = u^2\vec{i} + u\vec{j} + 2u\vec{k}$ and $\overline{B} = \vec{j} - u\vec{k}$ then $\frac{d}{du}\left(\overline{A} \cdot \overline{B}\right)$ is

   (a) $2u - 1$ (b) $2u + 1$

   (c) $1 - 4u$ (d) $1 + 4u$

2. If $\overline{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then $\nabla \times \overline{r}$ is

(a) 0 (b) 1

(c) 2 (d) 3.

3. $\int_0^1 \int_0^2 xy^2 \, dy \, dx =$

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) 1 (d) $\frac{4}{3}$

4. $\int_0^{\pi} \int_0^1 r^4 \sin\theta \, dr \, d\theta$

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) .1

5. If $\overline{f} = x^2\overline{i} - xy\overline{j}$ and C is the straight line joining the points (0, 0) and (1, 1) then $\int_C \overline{f} \cdot d\overline{r} =$

(a) 1 (b) 0

(c) –1 (d) 2

6. The value of $\iint dx\, dy$ over the region bounded by $x = 0,\ x = 2,\ y = 0;\ y = 2$ is

(a) 2 (b) 4

(c) 0 (d) 3

7. If R is any closed region of the xy-plane bounded by a simple closed curve C then $\int_C y\, dx + x\, dy$ is

(a) 1 (b) 0

(c) π (d) 2π

8. Green's theorem connects

(a) line integral and double integral

(b) line integral and surface integral

(c) double integral and surface integral

(d) surface integral and volume integral.

9. An example of an even function is

(a) x (b) $|x|$

(c) $x + x^3$ (d) $x + x^2$

10. The Fourier coefficient a_0 for the function $f(x) = x \sin x$ in $(0, 2\pi)$ is

(a) 0 (b) 1

(c) 2 (d) –2.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Find ϕ if

$$\nabla\phi = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}.$$

Or

(b) Prove that $curl\,(\bar{r} \times \bar{a}) = -2\bar{a}$, where $\bar{a}$ is a constant vector.

12. (a) Evaluate $\int_0^{\pi/2}\int_0^{\pi/2} \sin(x + 2y)\,dx\,dy$.

Or

(b) Evaluate $\int_0^{a}\int_0^{b}\int_0^{c} (x + y + z)\,dx\,dy\,dz$.

13. (a) Evaluate $\int_C \bar{f} \cdot d\bar{r}$, where

$\bar{f} = (2y + 3)\vec{i} + xz\vec{j} + (yz - x)\vec{k}$ and C is the straight line joining the points (0, 0, 0) and (2, 1, 1).

Or

(b) Evaluate $\iint_S \bar{f} \cdot \hat{n} dS$ where

$\bar{f} = (x + y^2)\vec{i} - 2x\vec{j} + 2yz\vec{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

14. (a) By using Stoke's theorem, prove that $\int_C \bar{r} \cdot d\bar{r} = 0$ where $\bar{r} = x\vec{i} + y\vec{j} + z\vec{k}$.

Or

(b) If $\bar{f} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ and V is the volume enclosed by the cube $0 \le x, y, z \le 1$ then evaluate $\iiint_V \nabla \cdot \bar{f}\, dV$.

15. (a) Find the Fourier series for the function

$$f(x) = \begin{cases} -x & -\pi \le x < 0 \\ x & 0 \le x \le \pi. \end{cases}$$

Or

(b) Find the Fourier sine series for the function $f(x) = k$ in the interval $0 < x < \pi$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Prove that $div\left(r^n \bar{r}\right) = (n+3)\,r^n$. Deduce that $r^n \bar{r}$ is solenoidal iff $n = -3$.

Or

(b) Prove that

$$curl\left(\bar{f} \times \bar{g}\right) = \left(\bar{g} \cdot \nabla\right)\bar{f} - \left(\bar{f} \cdot \nabla\right)\bar{g} + \bar{f}\, div\, \bar{g} - \bar{g}\, div\, \bar{f}$$

17. (a) Find the area of the circle $x^2 + y^2 = r^2$ by using double integral.

Or

(b) Evaluate $\iiint_D \frac{dx\, dy\, dz}{(x+y+z+1)^3}$ where D is the region bounded by the planes $x=0$; $y=0$; $z=0$ and $x+y+z=1$.

18. (a) Evaluate $\iint_S \left(\nabla \times \bar{f}\right) \cdot \hat{n}\, dS$ where $\bar{f} = y^2\bar{i} + y\bar{j} - xz\bar{k}$ and S is the upper half of the sphere $x^2+y^2+z^2=a^2$ and $z \geq 0$.

Or

(b) Find $\int_C \bar{f} \cdot d\bar{r}$ where $\bar{f} = 3x^2\vec{i} + (2xz - y)\vec{j} + z\vec{k}$ and C is

(i) the straight line from (0, 0, 0) to (2, 1, 3).

(ii) the curve $x = 2t^2$; $y = t$; $z = 4t^2 - 1$ from $t = 0$ to $t = 1$.

(iii) the curve $x^2 = 4y$; $3x^2 = 8z$ from $x = 0$ to $x = 2$.

19. (a) Verify Green's theorem for

$$\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy,$$

where C is the boundary of the region R enclosed by $x = 0$; $y = 0$; $x + y = 1$.

Or

(b) Verify Gauss divergence theorem for $\bar{f} = y\vec{i} + x\vec{j} + z^2\vec{k}$ for the cylindrical region S given by $x^2 + y^2 = a^2$; $z = 0$ and $z = 4$.

20. (a) Find the Fourier series for the function $f(x) = x^2$ in the interval $-\pi \le x \le \pi$ and deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$.

Or

(b) (i) Prove that the Fourier cosine series for the function $f(x)=x$ in the interval $0 \le x \le \pi$ is

$$x=\frac{\pi}{2}-\frac{4}{\pi}\left[\frac{\cos x}{1^2}+\frac{\cos 3x}{3^2}+\frac{\cos 5x}{5^2}+\cdots\right].$$

Hence deduce that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots=\frac{\pi^2}{8}.$$

(ii) Prove that the Fourier sine series for the function $f(x)=x$ in the interval $0 \le x \le \pi$ is

$$x=2\left[\frac{\sin x}{1}-\frac{\sin 2x}{2}+\frac{\sin 3x}{3}-\cdots\right].$$

Hence deduce that $1-\frac{1}{3}+\frac{1}{5}-\cdots=\frac{\pi}{4}$.

(7 pages) **Reg. No. :** ....

Code No. : 40572 E **Sub. Code : SMMA 52**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Fifth Semester

Mathematics — Core

REAL ANALYSIS - II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. In any metric space, the diameter of the empty set Φ is ______

(a) 0

(b) 1

(c) $-\infty$

(d) $+\infty$

2. If $A = \left\{0, 1, \frac{1}{2}, \ldots, \frac{1}{n}, \ldots\right\}$, then $\text{Int}\, A =$ ______

(a) 0 (b) 1

(c) $\frac{1}{n}$ (d) ϕ

3. In R with usual metric, $(a, b]$ is ______ interval

(a) an open

(b) a closed

(c) both open and closed

(d) neither open nor closed

4. The set of irrational number in R is ______

(a) open (b) closed

(c) dense (d) complete

5. If $f : (0, 1] \rightarrow R$ is a function defined by $f(x) = \frac{1}{x}$, then f in $(0, 1]$ is ______

(a) both continuous and uniformly continuous

(b) continuous but not uniformly continuous

(c) not continuous

(d) uniformly continuous

6. For any $n \in Z$, oscillation $w(f, n) =$ ________

(a) n (b) 0

(c) 1 (d) ∞

7. R means

(a) connected

(b) not connected

(c) compact

(d) neither connected nor compact

8. The set $\{[0, n) | n \in N\}$ is an open cover for ________

(a) R (b) N

(c) $[0, \infty)$ (d) $(-\infty, \infty)$

9. If f is differentiable at a point c, then f is ________

(a) continuous at c

(b) not continuous at c

(c) uniformly continuous at c

(d) both continuous and uniformly continuous at c

10. $S(P, f) = \sum_{K=1}^{n} f(t_k) \Delta x_k$ is called as ________

(a) Riemann-stieltjes sum

(b) Riemann-sum

(c) Riemann integral

(d) Riemann-stieltjes integral

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If (M, d) is a metric space and if $d_1(x, y) = \min\{1, d(x, y)\}$, then prove that d_1 is a metric on M.

Or

(b) Prove that in any metric space, the intersection of a finite number of open sets in open.

12. (a) Prove that in any metric space, every closed ball is a closed set.

Or

(b) A subset A of a complete metric space M is complete iff A is closed.

13. (a) Define a continuous functions. Prove that the composition of two continuous functions is continuous.

Or

(b) If f is monotonic on $[a, b]$, then prove that the set of discontinuities of f is countable.

14. (a) Prove that any continuous image of a connected set is connected.

Or

(b) Prove that a subset A of R is compact iff A is closed and bounded .

15. (a) If f and g are defined on (a, b) and differentiable at c, then prove that f/g is also differentiable at c if $g c \neq 0$ and find $(f/g)'(c)$.

Or

(b) State and prove the mean value theorem for derivatives.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) (i) If (M, d) is a metric space and if $d_1(x, y) = \dfrac{d(x, y)}{1 + d(x, y)}$, then prove that d_1 is a metric on M.

(ii) Prove that in any metric space (M, d) each open ball is an open set.

Or

(b) If (M, d) is metric space and if $A, B \leq M$, then prove the followings:

(i) $Int\ A$ = union of all open sets contained in A

(ii) $A \subseteq B \Rightarrow Int\ A \subseteq Int\ B$

(iii) $Int(A \cap B) = Int\ A \cap Int\ B$

(iv) $Int\ (A \cup B) \supseteq Int\ A \cup Int\ B$.

17. (a) Show that l_2 is complete.

Or

(b) State and prove Baire's category theorem.

18. (a) Prove that f is continous iff the inverse image of every open set is open.

Or

(b) Show that $f: R \to R$ is continuous at $a \in R$ iff $w(f, a) = 0$.

19. (a) Prove that a subspace of R is connected iff it is an interval.

Or

(b) State and prove Heine-Borel theorem.

20. (a) State and prove the chain rule for derivatives.

Or

(b) State and prove Taylor's theorem.

———————

(7 pages) **Reg. No. :** ....

Code No. : 41154 E **Sub. Code : JMMA 31/ JMMC 31/SMMA 31**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Third Semester

Mathematics/Mathematics with CA – Main

REAL ANALYSIS – I

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. If $x(y+z) = xy + xz$ is called
   (a) commutative property
   (b) associative property
   (c) distributive property
   (d) identity property

2. The set $R^{+} = (0, +\infty)$ is
   (a) bounded above
   (b) bounded below
   (c) unbounded above
   (d) unbounded below

3. The range of the sequence $(1-(-1)^n)$ is
   (a) $\{1, -1, 1, -1, \ldots\}$ (b) $\{0, 2\}$
   (c) $\{0, 1\}$ (d) $(-\infty, \infty)$

4. The following are true except ————.
   (a) $n^{\frac{1}{n}}$ is a bounded sequence
   (b) $n^{\frac{1}{n}}$ is a convergent sequence
   (c) $n^{\frac{1}{n}}$ is a divergent sequence
   (d) $\lim\limits_{n \to \infty} n^{\frac{1}{n}} = 1$

5. $\lim\limits_{n \to \infty} \left(1 + \frac{1}{n}\right)^n =$
   (a) 0 (b) 1
   (c) ∞ (d) e

6. Every bounded sequence has atleast ———— limit point.

(a) zero (b) one

(c) two (d) three

7. If $\sum_{n=1}^{\infty} a_n$ converges to S then

(a) $\lim_{n\to\infty} a_n = 0$ (b) $\lim_{n\to\infty} a_n = S$

(c) $\lim_{n\to\infty} a_n = 1$ (d) None

8. Example of a series $\sum_{n=1}^{\infty} a_n$ which is divergent but $\lim_{n\to\infty} a_n = 0$

(a) $\sum_{n=1}^{\infty} \frac{1}{n}$ (b) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$ (d) $\sum_{n=1}^{\infty} \frac{1}{n^3}$

9. If n^{th} term of a series $a_n = \frac{1\cdot 2\cdot 3 \ldots n}{3\cdot 5\cdot 7 \ldots (2n-1)}$ then $\lim_{n\to\infty} \frac{a_n}{a_{n+1}} =$ ————.

(a) 1 (b) 2

(c) $\frac{1}{2}$ (d) 0

10. For the Logarithmic series, the radius of convergence R is ————.

(a) 0

(b) 1

(c) ∞

(d) <1

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If a and b are real numbers such that $a \le b + \epsilon$ for every $\epsilon > 0$ then prove that $a \le b$.

Or

(b) If n is a positive integer which is not a perfect square, then show that $\sqrt{n}$ is irrational.

12. (a) Show that $\lim_{n\to\infty} \frac{\sin n}{n} = 0$.

Or

(b) Show that if $(a_n) \to 0$ and (b_n) is bounded then prove that $(a_n b_n) \to 0$.

13. (a) Show that any convergent sequence is a cauchy sequence.

Or

(b) Prove that $\lim_{n\to\infty}\frac{x^n}{n!}=0$.

14. (a) Discuss the convergence of the series $1+\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^4}+\dots$

Or

(b) State and prove D'Alembert's ratio test.

15. (a) Test for convergence of the series $\frac{1}{2^3}-\frac{1}{3^3}(1+2)+\frac{1}{4^3}(1+2+3)-\dots$

Or

(b) Find the radius of convergence for the exponential series.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Cauchy-Schwarz inequality.

Or

(b) Show that e is an irrational number.

17. (a) Show that $\lim_{n\to\infty}\left(a^{\frac{1}{n}}\right)=1$, where $a>0$ is any real number.

Or

(b) If $(a_n)\to a$ and $a_n\neq 0$ for all n and $a\neq 0$ then prove that $\left(\frac{1}{a_n}\right)\to\left(\frac{1}{a}\right)$.

18. (a) Discuss the behaviour of the geometric series.

Or

(b) Show that $\lim_{n\to\infty}\frac{\log n}{n^p}=0$, if $p>0$.

19. (a) Test the convergence of $\frac{1}{3}x+\frac{1}{3}.\frac{2}{5}x^2+\frac{1}{3}.\frac{2}{5}.\frac{3}{7}x^3+\dots$

Or

(b) State and prove Gauss test.

20. (a) Test for convergence of the series $\sum \frac{(-1)^n}{n^p}$.

Or

(b) State and prove Leibnitz's test.

(7 pages) **Reg. No. : ....**

Code No. : 41383 E **Sub. Code : SMMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Second Semester

Mathematics — Main

ANALYTICAL GEOMETRY OF THREE DIMENSIONS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. The equation of the tangent whose vertical angle ϕ is

   (a) $\frac{l}{r} = \rho \cos\theta - \cos(\theta - \phi)$

   (b) $\frac{l}{r} = \rho \cos\theta + \cos(\theta - \phi)$

   (c) $\frac{l}{r} = \rho \cos\theta - \sin(\theta - \phi)$

   (d) $\frac{l}{r} = \rho \cos\theta + \sin(\theta - \phi)$

2. The asymptotes of the conic $\frac{l}{r} = 1 + e\cos\theta$ is

(a) $\frac{e^2-1}{e}\left\{\cos\theta \pm \frac{\sin\theta}{\sqrt{e^2-1}}\right\}$

(b) $\frac{e^2-1}{e}\left\{\sin\theta \pm \frac{\sin\theta}{\sqrt{e^2-1}}\right\}$

(c) $\frac{e^2+1}{e}\left\{\cos\theta \pm \frac{\sin\theta}{\sqrt{e^2-1}}\right\}$

(d) None

3. $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n} = r$ is the equation of the ————.

(a) Circle (b) Straight line

(c) Ellipse (d) Hyperbola

4. $x^2 + y^2 + z^2 - 2x + 6y + 4z - 35 = 0$ is then the centre is, ————.

(a) $(-1, 3, 2)$ (b) $(1, -3, -2)$

(c) $(-2, 6, 4)$ (d) $(2, -6, -4)$

5. When a sphere is a cut by a plane through its centre ———— is obtained.

(a) Circle (b) Ellipse

(c) Parabola (d) Hyperbola

6. A cylinder is a surface generated by ————.

(a) Straight line (b) Sphere

(c) Circle (d) None

7. The fixed distance of the right circular cylinder is

(a) Semi latus rectum (b) Radius

(c) Axis (d) None

8. The constant angle of right circular cone is

(a) acute angle (b) semi vertical

(c) right angle (d) none

9. Equation of hyperbolic of one sheet is

(a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ (b) $\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

(c) $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ (d) None

10. The intersection of $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ————.

(a) a parabola (b) an ellipse

(c) a hyperbola (d) a circle

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Show that the points $(5, 3, -2)$, $(3, 2, 1)$ and $(-1, 0, 7)$ are collinear.

Or

(b) Find the angle between two diagonals of a cube.

12. (a) Derive the equation of the plane passing through the points (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3).

Or

(b) Find the distance between the parallel planes $2x - 2y - z = 3$ and $4x - 4y + 2z + 5 = 0$.

13. (a) Find the equation of the image of the line $\frac{x-1}{2} = \frac{y+2}{-5} = \frac{z-3}{2}$ in the plane $2x - 3y + 3 + 2z = 0$.

Or

(b) Find the shortest distance between the lines $\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$; $\frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$.

14. (a) A sphere of constant radius k passes through the origin and meets the axes in A, B, C. Prove that the centroid of the triangle ABC lies on the sphere $9(x^2 + y^2 + z^2) = 4k^2$.

Or

(b) Show that the plane $2x - y - 2z = 16$ touches the sphere $x^2 + y^2 + z^2 - 4x + 2y - 2z - 3 = 0$ and find the point of contact.

15. (a) Find the equation of the cone of the second degree with passes through the axes.

Or

(b) Find the equation of a right circular cylinder of radius 3 with axis $\frac{x+2}{3} = \frac{y-4}{6} = \frac{z-1}{2}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Show that the four points $(4, -1, 2)$, $(0, -2, 3)$, $(1, -5, -1)$ and $(2, 0, 1)$ lie on a sphere whose centre is $(2, -3, 1)$. Find the radius of the sphere.

Or

(b) Show that the straight lines whose direction cosines are given by $al + bm + cn = 0$, $fmn + gnl + hlm = 0$ are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ and parallel if $\sqrt{af} + \sqrt{hg} + \sqrt{ch} = 0$.

17. (a) Show that the following points are coplanar and find the equation of the plane on which they lie

(i) $(0, -1, -1)$, $(-4, 4, 4)$, $(4, 5, 1)$ and $(3, 9, 4)$

(ii) $(0, 2, -4)$, $(-1, 1, -2)$, $(-2, 3, 3)$ and $(-3, -2, 1)$.

Or

(b) Show that the equation

$$x^2 + y^2 + 4z^2 + 4yz + 4zx + 2xy + 7(x + y + 2z) + 12 = 0$$

represents a pair of parallel planes and find the distance between them.

18. (a) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the axes in A, B, C. Find the coordinates of the orthocentre of the triangle ABC.

Or

(b) Prove that the lines $\frac{x+1}{-3}=\frac{y+10}{8}=\frac{z-1}{2}$; $\frac{x+3}{-4}=\frac{y+1}{7}=\frac{z-4}{1}$ are coplanar. Also their points intersection and the plane through them.

19. (a) A plane passes through a fixed point (a, b, c) and cuts the axes A, B, C show that the locus of the centre of the sphere $OABC$ is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$.

Or

(b) The plane ABC, whose equation is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the axes in A, B, C. Find the equation to the circumcircle of the triangle ABC and obtain the coordinates of its centre and radius.

20. (a) Find the condition for the equation

$$F(x, y, z)=ax^2+by^2+cz^2+2fyz+2gzx+2hxy+2ux+2vy+2wz+d=0$$

to represent a cone.

Or

(b) Find the equation of the right circular cylinder described on the circle through the points $(a, 0, 0)$, $(0, a, 0)$, $(0, 0, a)$ as a guiding curve.

(7 pages) **Reg. No. :** ....

Code No. : 40331 E **Sub. Code : JMMA 11/ JMMC 11**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

First Semester

Mathematics/ Mathematics with CA – Main

CALCULUS

(For those who joined in July 2016 only)

Time : Three hours Maximum : 75 marks

PART A — $(10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. The locus of the centre of curvature for a curve is ——————.

   (a) Involute (b) Evolute

   (c) Complete (d) Regular

2. If $u = x^3 + y^3 + 3x^2y + 3xy^2$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

(a) u (b) $2u$

(c) $3u$ (d) $-u$

3. Pedal equation of the circle with radius a is

(a) $\rho = \frac{r^2}{a}$ (b) $\rho = \frac{r^2}{a^2}$

(c) $\rho^2 = \frac{r}{a}$ (d) None

4. General equation of the asymptotes of a hyperbola is

(a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (b) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(c) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 0$ (d) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

5. $\int_0^{\frac{\pi}{2}} \sin^5 x\, dx =$

(a) $\frac{8}{15}$ (b) $\frac{4}{15}$

(c) $\frac{8\pi}{15}$ (d) $\frac{4\pi}{15}$

6. $\int_0^1\int_0^2 xy\, dx\, dy =$

(a) 0 (b) $\frac{1}{6}$

(c) 1 (d) $\frac{3}{2}$

7. If $x = u(1+v), y = v(1+u)$ then $\frac{\partial(x,y)}{\partial(u,v)} =$ ________.

(a) $1+u+v$ (b) $u+v$

(c) $1-u+v$ (d) $1-u-v$

8. Change the order of integration $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y} dx\, dy$

(a) $\int_0^1\int_x^y \frac{e^{-y}}{y} dx\, dy$ (b) $\int_0^\infty\int_0^x \frac{e^{-y}}{y} dx\, dy$

(c) $\int_0^1\int_0^y \frac{e^{-y}}{y} dx\, dy$ (d) None of these

9. $\left[\Gamma\left(\frac{1}{2}\right)\right]^2 =$

(a) π (b) $\frac{\pi}{2}$

(c) $\sqrt{\pi}$ (d) $\frac{\sqrt{\pi}}{2}$

10. $\beta(2, 3) =$ ————.

(a) $\frac{1}{2}$ (b) $\frac{5\pi}{32}$

(c) $\frac{1}{12}$ (d) $\frac{3}{8}$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Derive the Cartesian formula for the radius of curvature.

Or

(b) Find the radius of curvature of the Cardioid $r = a(1 - \cos\theta)$.

12. (a) Prove that the $p-r$ equation of the cardioid $r = a(1-\cos\theta)$ is $\rho^2 = \frac{r^3}{2a}$.

Or

(b) Find the asymptote of $x^3 + y^3 = 3axy$.

13. (a) Show that $x^4 - 2x^2y - xy^2 - 2x^2 - 2xy + y^2 - x + 2y + 1 = 0$ has a single cusp of the second kind at (0, –1).

Or

(b) Trace the curve $y = \frac{x}{(2-x)^2}$.

14. (a) Evaluate $\iint (x^2 + y^2)\,dx\,dy$ over the region for which x, y are each ≥ 0 and $x + y \leq 1$.

Or

(b) If $x + y = u$, $y = uv$, change the variables to u, v in the integral $\iint [xy(1-x-y)]^{\frac{1}{2}}\,dx\,dy$ taken over the area of the triangle with sides $x = 0, y = 0$, $x + y = 1$ and evaluate it.

15. (a) Show that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$.

Or

(b) Evaluate $\int_0^1 x^m \left(\log\frac{1}{x}\right)^n dx$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Prove that the radius of curvature at a point $\left(a\cos^3\theta, a\sin^3\theta\right)$ on the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ is $3a\sin\theta\cos\theta$.

Or

(b) Find the evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

17. (a) Find the $p-r$ equation of the curve $x^2+y^2=ax$ and deduce its radius of curvature.

Or

(b) Find the rectilinear asymtotes of $2x^4 - 5x^2y^2 + 3y^4 + 4y^3 - 6y^3 + x^2 + y^2 - 2xy + 1 = 0$.

18. (a) Examine for double points of the curve $x^4 - 2ay^3 - 3a^2y^2 - 2a^2x^2 + a^4 = 0$.

Or

(b) Trace the curve $x^3 + y^3 = 3axy$.

19. (a) Change the order of integration in the integral $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\, dx\, dy$ and evaluate it.

Or

(b) Evaluate $\iiint xyz\, dx\, dy\, dz$ over the +ve octant of the sphere $x^2 + y^2 + z^2 = a^2$ by transformation into spherical coordinates.

20. (a) Prove that $\beta(m,n) = \dfrac{\Gamma m \Gamma n}{\Gamma(m+n)}$.

Or

(b) Prove that $\int_0^\infty x^2 e^{-x^8} dx \times \int_0^\infty x^2 e^{-x^4} dx = \dfrac{\pi}{16\sqrt{2}}$.

(7 pages) **Reg. No. :** ....

Code No. : 41162 E **Sub. Code : JMMA 64/ JMMC 64**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Sixth Semester

Mathematics/Mathematics with CA – Main

GRAPH THEORY

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The number of edges in K_6 is ————.

(a) 36 (b) 30

(c) 15 (d) 12

2. Which of the following is not true?

(a) Every walk is a path

(b) Every path is a trail

(c) Every trail is a walk

(d) Every path is a walk

3. If we remove the cut vertices from a graph G, then the number of components

(a) decreases

(b) increases

(c) no change

(d) nothing can be said

4. The number of edges in a tree with 20 vertices is

(a) 20 (b) 21

(c) $\frac{20 \times 19}{2}$ (d) 19

5. The smallest nm planar complete graph is

(a) K_3 (b) K_4

(c) K_5 (d) K_6

6. If G is a (p, q) plannan graph with f faces, then $p - q + f =$ ________.

(a) 1 (b) 2

(c) 3 (d) 4

7. The rank of cut set matrix with p vertices is ________.

(a) $\leq p-1$ (b) $= p-1$

(c) $\geq p-1$ (d) $= p+1$

8. The $(i,i)^{th}$ entry of the adjauncy matrix of a graph ________

(a) 1 (b) 0

(c) 2 (d) –1

9. Chromatic number of $K_{m,n}$ is ________.

(a) p (b) $p-1$

(c) 1 (d) 2

10. Chromatic polynomial $f(K_1, \lambda) =$ ________.

(a) λ (b) λ^2

(c) $\lambda^2 + 1$ (d) $\lambda^2 + \lambda$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that the sum of the degrees of the points of any graph is twice the number of lines.

Or

(b) A graph G is connected iff for any partition of V into subsets V_1 and V_2 there is a line of G joining a point of V_1 to a point of V_2.

12. (a) Prove that every tree has either one or two centers.

Or

(b) Prove that every connected graph has a spanning tree.

13. (a) Prove that K_5 is non-planon.

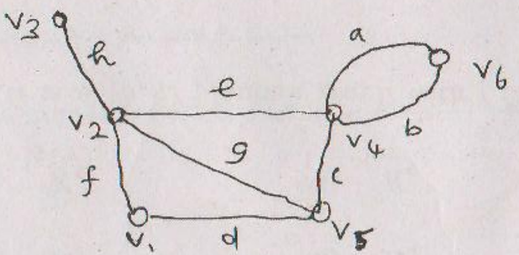
Or

(b) If G is a (p, q) planon graph in which every face is an n cycle, prove that $q = \frac{n(p-2)}{n-2}$.

14. (a) Let G_1 be a (p_1, q_2) graph and G_2 be (p_2, q_2) graph. Then prove that $G_1 \times G_2$ is a $(p_1 p_2, q_1 p_2 + q_2 p_1)$ graph.

Or

(b) Find the incidence matrix for the given graph.



15. (a) If G is a tree with $n \geq 2$ points, prove that the chromatic polynomial $f(G, \lambda) = \lambda(\lambda_{-1})^{n-1}$.

Or

(b) Prove that every k-chromatic graph has atleast k vertices of degree atleast $k-1$.

PART C — $(5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Dirac's theorem.

Or

(b) Prove that a simple graph with p vertices and k components can have atmost $\frac{(p-k)(p-k+1)}{2}$ edges.

17. (a) Prove that a connected graph has p vertices and $p-1$ edges iff it is a tree.

Or

(b) For any graph G, prove that vertex connectivity $\leq$ line connectivity $\leq \delta$.

18. (a) Prove that a connected planar graph with p vertices and q edges has $q - p + 2$ regions.

Or

(b) Write down the relationship between the planar graph and its dual.

19. (a) Write remarks on adjacency matrix.

Or

(b) Prove that the rank of cut set matrix $C(G)$ = the rank of incidence matrix $A(G)$ = rank of graph G.

20. (a) State and prove Five colour theorem.

Or

(b) If d_{max} is the maximum degree of the vertices in a graph G, then prove that chromatic number of $G \leq 1 + d_{max}$.

(7 pages) **Reg. No. :** ....

Code No. : 41153 E **Sub. Code : JMMA 22/ JMMC 22/SMMA 22**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Second Semester

Mathematics/Mathematics with C.A. — Main

DIFFERENTIAL EQUATIONS

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The solution of $\frac{dy}{dx}+\left\{\frac{1-y^2}{1-x^2}\right\}^{1/2}=0$ is

   (a) $x^2y^2=c$

   (b) $\sin^{-1}x+\sin^{-1}y=c$

   (c) $\tan^{-1}x+\tan^{-1}y=c$

   (d) $\tan^{-1}x\ \tan^{-1}y=c$

2. If $\frac{dx}{dt} = -kx\,(k > 0)$, k is called as ————.

(a) Rate increase (b) Rate decrease

(c) Rate constant (d) Constant

3. $I =$ ————.

(a) $\frac{dQ}{dt}$ (b) $\frac{dI}{dt}$

(c) $\frac{L\,dI}{dt}$ (d) $\frac{1}{c}Q$

4. Solution of $y = xp + p^2$ is

(a) $y = c$ (b) $p = c$

(c) $y = cx + c^2$ (d) $y = c^2$

5. If $z = \frac{1}{D-\alpha}X$ then $z =$ ————.

(a) $\int Xe^{-2x}\,dx$ (b) $\int Xe^{2x}\,dx$

(c) $e^{-2x}\int Xe^{-2x}\,dx$ (d) $e^{2x}\int Xe^{-2x}\,dx$

6. $\frac{1}{D^2 + a^2}\cos ax =$ ————.

(a) $\frac{x\sin ax}{2a}$ (b) $\frac{-x\sin ax}{2a}$

(c) $\frac{x\cos ax}{2a}$ (d) $\frac{-x\cos ax}{2a}$

7. If $\theta = x\frac{d}{dx}$ then $\theta^r x^m =$ ________.

(a) x^m (b) $m!$

(c) 0 (d) $m^r x^m$

8. $D^n(e^{ax}v) =$ ________.

(a) $e^{ax}v$ (b) $ne^{ax}v$

(c) $e^{ax}(D+a)^n v$ (d) $(D+a)^n v$

9. If the number of constants to be eliminated is equal to the number of independent variables an equation of ________ results.

(a) first order

(b) second order

(c) more than second order

(d) none of these

10. The complete integral of $z = px + qy + pq$ is

(a) $z = ax + by + c$ (b) $z = ax + by + ab$

(c) $z = xy$ (d) none of these

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Solve : $p^2 + \left(x + y - \frac{2y}{x}\right) p + xy + \frac{y^2}{x^2} - y - {y^2}/{x} = 0$.

Or

(b) Solve : $x^2 = (1 + p^2)$.

12. (a) Solve : $(D^2 - 4D + 3) = \sin 3x \cos x$.

Or

(b) Solve : $(D^2 + 1)\ y = x^2 + 1$.

13. (a) Explain the method of solving linear equations with variable coefficients.

Or

(b) Solve : $x^2 \frac{d^2y}{dx} + x\frac{dy}{dx} + 2y = x^2$.

14. (a) Solve : $p^2 + q^2 = npq$.

Or

(b) Eliminate f and φ from the relation $z = f(x + ay) + \varphi(x - ay)$.

15. (a) State Ohm's law and Kirchiff's law.

Or

(b) A substance doubles itself in three hours, by what ratio will it increase in 15 hours, on the assumption that the quantity increases at a rate proportional to itself.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Solve :

(i) $p^2 y + p(x - y) - x = 0$

(ii) $xp(3y^2 - ax) = y(2y^2 - ax)$.

Or

(b) Solve : $\dfrac{dx}{dt} = ax + by + c$; $\dfrac{dy}{dt} = a'x + b'y + c'$.

17. (a) Solve : $(D^4 + D^3 + D^2)y = 5x^2 + \cos x$.

Or

(b) Solve : $(D^2 - 2D + 4)y = e^x \sin x$.

18. (a) Solve :

$$x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \frac{\log x \ \sin(\cos x) + 1}{x}.$$

Or

(b) Solve :

$$(5+2x)^2 \frac{d^2y}{dx^2} - 6(5+2x)\frac{dy}{dx} + 8y = 6x.$$

19. (a) Solve :

$$(x^2 - a^2)\,p + (xy - a^2 \tan\alpha)\,q = x^2 - ay\cot\alpha.$$

Or

(b) Find the integral surface of $x^2 p + y^2 q + z^2 = 0$ which passes through the hyperbola $xy = x + y;\ z = 1$.

20. (a) In the circuit described by $L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + Q/C = E$ show that

(i) Ohm's law is satisfied whenever the current is a maximum of a minimum; and

(ii) The e.m.f. is increasing when the current is at a maximum and decreasing when it is at a maximum.

Or

(b) A tank contains 1,000 litres of brine in which 400 grams of salt are dissolved. Fresh water runs into the tank at the rate of 8 litres per minute and the mixture kept uniform by continuous stirring runs out at the same rate. How long will it be before only 300 grams of salt are left in the tank?

(8 pages) **Reg. No. :** ....

Code No. : 40345 E **Sub. Code : JAMA 21/ SAMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Second/Fourth Semester

Mathematics — Allied

VECTOR CALCULUS AND FOURIER SERIES

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $\overline{A} = u^2\vec{i} + u\vec{j} + 2u\vec{k}$ and $\overline{B} = \vec{j} - u\vec{k}$ then $\frac{d}{du}\left(\overline{A}\cdot\overline{B}\right)$ is

   (a) $2u-1$ (b) $2u+1$

   (c) $1-4u$ (d) $1+4u$

2. If $\bar{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then $\nabla \times \bar{r}$ is

(a) 0 (b) 1

(c) 2 (d) 3.

3. $\int_0^1 \int_0^2 xy^2 \, dy \, dx =$

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) 1 (d) $\frac{4}{3}$

4. $\int_0^\pi \int_0^1 r^4 \sin\theta \, dr \, d\theta$

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) .1

5. If $\bar{f} = x^2\vec{i} - xy\vec{j}$ and C is the straight line joining the points (0, 0) and (1, 1) then $\int_C \bar{f} \cdot d\bar{r} =$

(a) 1 (b) 0

(c) –1 (d) 2

6. The value of $\iint dx\,dy$ over the region bounded by $x=0,\ x=2,\ y=0;\ y=2$ is

(a) 2

(b) 4

(c) 0

(d) 3

7. If R is any closed region of the xy-plane bounded by a simple closed curve C then $\int_C y\,dx + x\,dy$ is

(a) 1

(b) 0

(c) π

(d) 2π

8. Green's theorem connects

(a) line integral and double integral

(b) line integral and surface integral

(c) double integral and surface integral

(d) surface integral and volume integral.

9. An example of an even function is

(a) x

(b) $|x|$

(c) $x + x^3$

(d) $x + x^2$

10. The Fourier coefficient a_0 for the function $f(x) = x \sin x$ in $(0, 2\pi)$ is

(a) 0 (b) 1

(c) 2 (d) –2.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Find ϕ if

$$\nabla \phi = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}.$$

Or

(b) Prove that $curl\,(\overline{r} \times \overline{a}) = -2\overline{a}$, where $\overline{a}$ is a constant vector.

12. (a) Evaluate $\int_0^{\pi/2}\int_0^{\pi/2} \sin(x + 2y)\, dx\, dy$.

Or

(b) Evaluate $\int_0^a\int_0^b\int_0^c (x + y + z)\, dx\, dy\, dz$.

13. (a) Evaluate $\int_C \overline{f} \cdot d\overline{r}$, where

$\overline{f} = (2y + 3)\vec{i} + xz\vec{j} + (yz - x)\vec{k}$ and C is the straight line joining the points (0, 0, 0) and (2, 1, 1).

Or

(b) Evaluate $\iint_S \bar{f} \cdot \hat{n} dS$ where

$\bar{f} = (x + y^2)\bar{i} - 2x\bar{j} + 2yz\bar{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

14. (a) By using Stoke's theorem, prove that $\int_C \bar{r} \cdot d\bar{r} = 0$ where $\bar{r} = x\vec{i} + y\vec{j} + z\vec{k}$.

Or

(b) If $\bar{f} = x^2\bar{i} + y^2\bar{j} + z^2\bar{k}$ and V is the volume enclosed by the cube $0 \le x, y, z \le 1$ then evaluate $\iiint_V \nabla \cdot \bar{f} \, dV$.

15. (a) Find the Fourier series for the function

$$f(x) = \begin{cases} -x & -\pi \le x < 0 \\ x & 0 \le x \le \pi. \end{cases}$$

Or

(b) Find the Fourier sine series for the function $f(x) = k$ in the interval $0 < x < \pi$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Prove that $div\left(r^n\bar{r}\right) = (n+3)\, r^n$. Deduce that $r^n\bar{r}$ is solenoidal iff $n = -3$.

Or

(b) Prove that

$$curl\left(\bar{f} \times \bar{g}\right) = \left(\bar{g} \cdot \nabla\right)\bar{f} - \left(\bar{f} \cdot \nabla\right)\bar{g} + \bar{f}\, div\, \bar{g} - \bar{g}\, div\, \bar{f}$$

17. (a) Find the area of the circle $x^2 + y^2 = r^2$ by using double integral.

Or

(b) Evaluate $\iiint\limits_D \dfrac{dx\, dy\, dz}{(x+y+z+1)^3}$ where D is the region bounded by the planes $x=0$; $y=0$; $z=0$ and $x+y+z=1$.

18. (a) Evaluate $\iint\limits_S \left(\nabla \times \bar{f}\right) \cdot \hat{n}\, dS$ where $\bar{f} = y^2\vec{i} + y\vec{j} - xz\vec{k}$ and S is the upper half of the sphere $x^2+y^2+z^2=a^2$ and $z \geq 0$.

Or

(b) Find $\int_C \bar{f} \cdot d\bar{r}$ where $\bar{f} = 3x^2\vec{i} + (2xz - y)\vec{j} + z\vec{k}$ and C is

(i) the straight line from (0, 0, 0) to (2, 1, 3).

(ii) the curve $x = 2t^2$; $y = t$; $z = 4t^2 - 1$ from $t = 0$ to $t = 1$.

(iii) the curve $x^2 = 4y$; $3x^2 = 8z$ from $x = 0$ to $x = 2$.

19. (a) Verify Green's theorem for

$$\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy,$$

where C is the boundary of the region R enclosed by $x = 0$; $y = 0$; $x + y = 1$.

Or

(b) Verify Gauss divergence theorem for $\bar{f} = y\bar{i} + x\bar{j} + z^2\bar{k}$ for the cylindrical region S given by $x^2 + y^2 = a^2$; $z = 0$ and $z = 4$.

20. (a) Find the Fourier series for the function $f(x) = x^2$ in the interval $-\pi \le x \le \pi$ and deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$.

Or

(b) (i) Prove that the Fourier cosine series for the function $f(x)=x$ in the interval $0 \leq x \leq \pi$ is

$$x=\frac{\pi}{2}-\frac{4}{\pi}\left[\frac{\cos x}{1^2}+\frac{\cos 3x}{3^2}+\frac{\cos 5x}{5^2}+\cdots\right].$$

Hence deduce that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots=\frac{\pi^2}{8}.$$

(ii) Prove that the Fourier sine series for the function $f(x)=x$ in the interval $0 \leq x \leq \pi$ is

$$x=2\left[\frac{\sin x}{1}-\frac{\sin 2x}{2}+\frac{\sin 3x}{3}-\cdots\right].$$

Hence deduce that $1-\frac{1}{3}+\frac{1}{5}-\cdots=\frac{\pi}{4}$.

(7 pages) **Reg. No. : ....**

Code No. : 41384 E Sub. Code : SMMA 41

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Fourth Semester

Mathematics – Main

ABSTRACT ALGEBRA – I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. Order of a non-zero element in $(Z,+)$ is ______________.

 (a) α (b) 0

 (c) 1 (d) 2

2. A group of order 12 cannot have a subgroup of order

 (a) 3 (b) 4

 (c) 5 (d) 6

3. If H and K are two finite subgroups of a group G the $[HK] =$

(a) $\frac{|K|}{|H \cap K|}$

(b) $\frac{|H||K|}{|H \cap K|}$

(c) $\frac{|H|}{|H \cap K|}$

(d) $\frac{|H| + |K|}{|H \cap K|}$

4. Let p be a prime number and a be any integer relatively prime to p. Then $a^{p-1} \equiv 1 (\text{mod } p)$

(a) Lagrange's theorem

(b) Fermat's theorem

(c) Euler's theorem

(d) Cauchy's theorem

5. The Kennel of a homomorphism $f : G \to G'$ is ________.

(a) a subgroup of G'

(b) a normal subgroup of G'

(c) a normal subgroup of G

(d) $\{e\}$

6. $f:(R^*,\cdot)\to(R^+,\cdot)$ defined by $f(x)=|x|$ is

(a) one-one

(b) homomorphism

(c) onto

(d) (b) and (c)

7. The map $f:Z\to z$ defined by $f(x)=x^2+3$ is __________.

(a) a ring homomorphism

(b) not a ring homomorphism

(c) a ring-isomorphism

(d) a ring ephimorphism

8. An example of an infinite commutative ring without identity is __________.

(a) $(Z,+,\cdot)$

(b) $(Z_n,\oplus,\otimes)$

(c) $(2Z,+,\cdot)$

(d) $M_2(R)$

9. The product of the polynomials $2x+4$ and $4x^2+3x+1$ in $Z_5[x]$ is __________.

(a) $3x^3+2x^2+4x+4$

(b) $8x^2+2x^2+4x+4$

(c) $8x^3+22x^2+14x+4$

(d) $3x^3+2x^2+3x+4$

10. Let $f(x)$, $g(x) \in Z_4[x]$ be defined as $f(x) = x^2 + 2x + 3$ and $g(x) = 3x^2 + 2x$ then degree of $[f(x) + g(x)] =$ ————————.

(a) 0 (b) 2

(c) 4 (d) 1

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Let G denote the set of all matrices of the form $\begin{pmatrix} x & x \\ x & x \end{pmatrix}$ where $x \in R$. Then prove that G is a group under matrix multiplication.

Or

(b) Prove that a non-empty subset H of a group G is a subgroup of G iff $a, b \in H \Rightarrow ab^{-1} \in H$.

12. (a) Let G be a group and $a, b \in G$ and then prove that

(i) order of a = order of a^{-1}

(ii) order of a = order of $b^{-1}ab$.

Or

(b) State and prove Fermat's theorem.

13. (a) Prove that any permutation can be expressed as a product of disjoint cycles.

Or

(b) $I(G)$ is a normal subgroup of $A(G)$ prove.

14. (a) Prove that the set of all real numbers of the form $a+b\sqrt{2}$ where $a,b \in Q$ under usual addition and multiplication is a ring.

Or

(b) Prove that a finite commutative ring R without zero-divisors is a field.

15. (a) Show that the homomorphic image of an internal domain need not be an integral domain.

Or

(b) Prove that $R[x]$ is an integral domain iff R is an integral domain.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Let H and K be two subgroups of a group G. Then prove that HK is a subgroup of G iff $HK = KH$.

Or

(b) If n is a prime number then prove that $Z_n - [0]$ is a group under multiplication modulo n.

17. (a) Prove that a subgroup of cyclic group is cyclic.

Or

(b) State and prove Lagrange's theorem.

18. (a) State and prove Cayley's theorem.

Or

(b) State and prove fundamental theorem of Homomorphism.

19. (a) Prove that any finite integral domain is a field.

Or

(b) Prove the characteristic of any field is either 0 or a prime number.

20. (a) Prove that any integral domain D can be embedded in a field F.

Or

(b) Let R be a ring and I be a subgroup of $(R, +)$. Prove that the multiplication in R/I given by $(I+a)(I+b)=I+ab$ is well defined if and only if I is an ideal of R.

(6 pages) **Reg. No. :** ....

Code No. : 41156 E **Sub. Code : JMMA 51/ JMMC 51**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Fifth Semester

Mathematics/Mathematics with CA — Main

REAL ANALYSIS — II

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. In any metric space, $d(\phi) = ?$

   (a) ∞ (b) $-\infty$

   (c) 1 (d) 0

2. If A is a finite subset of R, then $Int\ A =$

   (a) A (b) R

   (c) $R - \{0\}$ (d) ϕ

3. In R with usual metric, which of the following is a closed set?

   (a) $\left\{0, \frac{1}{2}, 1\right\}$ (b) $(0, 1]$

   (c) Q (d) $R - Q$

4. $D(Q) =$ ________.

   (a) Q (b) ϕ

   (c) $R - Q$ (d) R

5. If f is a continuous real valued function on a metric space M and if $A = \{x \in M / f(x) \geq 0\}$, then A is

   (a) an open set (b) a closed set

   (c) an empty set (d) a dense set

6. If $f : R \to R$ is continuous at 'a' then $w(f, a)$ is

   (a) 0 (b) 1

   (c) a (d) ∞

7. Which one of the following subsets of R is connected?

   (a) Q (b) Z

   (c) $R - \{0\}$ (d) $(0, 1)$

8. A compact subset of R is

(a) $[0, \infty)$ (b) $(0, 1)$

(c) $(1, 2)$ (d) $[1, 2]$

9. $\int_{-1}^{1} |x|\, dx =$ ________.

(a) 1 (b) $\frac{1}{2}$

(c) 0 (d) 2

10. If a function f is bounded and integrable on $[a, b]$, then $\lim_{n\to\infty} \int_a^b f(x) \cos nx\, dx$ is

(a) $= 0$ (b) > 0

(c) < 0 (d) ≤ 0

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let (M, d) be a metric space. Define $d_1(x, y) = \min\{1, d(x, y)\}$. Prove that d_1 is a metric on M.

Or

(b) Prove that in a metric space (M, d), each open ball is an open set.

12. (a) Prove that for any subset A of a metric space, $d(A) = d(\overline{A})$, where $d(A)$ is the diameter of A.

Or

(b) Prove that in any metric space, arbitrary intersection of closed sets is closed.

13. (a) Prove that the function $f : (0, 1) \to R$ defined by $f(x) = \frac{1}{x}$ is not uniformly continuous.

Or

(b) Let f, g be continuous real valued functions on a metric space M and $A = \{x / x \in M$ and $f(x) < g(x)\}$. Prove that A is open.

14. (a) If A, B are connected sets and $A \cap B \neq \phi$, then prove that $A \cup B$ is connected.

Or

(b) Prove that the continuous image of a compact metric space is compact.

15. (a) State and prove Abel's lemma.

Or

(b) Show that x^2 is integrable on any interval $[0, k]$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Let (M, d) be a metric space. Define $d_1(x, y) = \frac{d(x, y)}{1 + d(x, y)}$ $\forall\, x, y \in M$. Prove that d_1 is also a metric on M.

Or

(b) If we define $\rho(x, y) = 2d(x, y)$ in a metric space (M, d), then prove that d and ρ are equivalent metrics.

17. (a) Show that R^n with usual metric is complete.

Or

(b) State and prove Baire's category theorem.

18. (a) Prove that f is continuous if and only if inverse image of every open set is open.

Or

(b) Prove that $f : M_1 \to M_2$ is continuous if and only if $f(\overline{A}) \subseteq \overline{f(A)}$ $\forall\, A \subseteq M_1$.

19. (a) Prove that a subspace of R is connected if and only if it is an interval.

Or

(b) State and prove Heine Borel theorem.

20. (a) (i) State and prove First mean value theorem.

(ii) State and prove Generalised mean value theorem.

Or

(b) If a function f is continuous on $[0, 1]$ show that $\lim_{n \to \infty} \int_0^1 \frac{nf(x)}{1 + n^2x^2}\, dx = \frac{\pi}{2} f(0)$.

(8 pages) **Reg. No. : ....**

Code No. : 40014 E **Sub. Code : GAMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Second/Fourth Semester

Mathematics – Allied

VECTOR CALCULUS

(For those who joined in July 2012 – 2015)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ then $\nabla \cdot \bar{r} =$ ————.

   (a) 1

   (b) 0

   (c) 3

   (d) $x^2 + y^2 + z^2$

2. The vector function $f = x^2\bar{i} + y^2\bar{j} + z^2\bar{k}$ is ________.

(a) solenoidal

(b) irrotational

(c) harmonic

(d) neither solenoidal nor irrotational

3. $\int e^{ax+b}dx =$ ________.

(a) $\frac{1}{b}e^{ax+b}$

(b) $\frac{1}{a}e^{ax+b}$

(c) $\frac{1}{a}e^{bx+a}$

(d) $\frac{1}{b}e^{bx+a}$

4. $\int \cot\theta \, d\theta =$ ________.

(a) $\log \sin\theta$

(b) $\log \tan\theta$

(c) $\tan\theta$

(d) $\sin\theta$

5. The value of $\iint dx\,dy$ over the region bounded by $x = 0, x = 2, y = 0, y = 2$, is ________.

(a) 2

(b) 4

(c) 0

(d) 3

6. The value of $\int_0^a \int_0^a \int_0^a dz\,dy\,dx$ is ________.

(a) a^3

(b) a^2

(c) a

(d) 1

7. If C is the straight line joining $(0, 0, 0)$ and $(1, 1, 1)$ then $\int_C \bar{r} \cdot dr$ is ________.

(a) $\frac{1}{2}$

(b) 1

(c) $\frac{3}{2}$

(d) 2

8. If $\bar{f} = (x^2 + y^2)\bar{i} + (x^2 - y^2)\bar{j}$ then the value of $\int_C r \cdot dr$ where C is the part of the curve $y = x^2$ joining the points $(0, 0)$ and $(1, 1)$ is ________.

(a) 0

(b) $\frac{9}{10}$

(c) $\frac{1}{2}$

(d) 2

9. If V is the volume enclosed by the closed surface S then the value of $\iint_S \overline{r} \cdot \overline{n}\, dS$ is ————.

(a) $3V^2$ (b) $3V$

(c) $6V$ (d) 0

10. Gauss's divergence theorem connects

(a) line integral and double integral

(b) line integral and surface integral

(c) double integral and surface integral

(d) surface integral and volume integral

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $\overline{r} = \overline{a}\cos\omega t + \overline{b}\sin\omega t$ where $\overline{a}, \overline{b}$ are constant vectors and ω is a constant prove that $\overline{r} \times \frac{d\overline{r}}{dt} = \omega(\overline{a} \times \overline{b})$ and $\frac{d^2\overline{r}}{dt^2} + \omega^2\overline{r} = 0$.

Or

(b) Show that $div\left(\frac{\overline{r}}{r}\right) = \frac{2}{r}$.

12. (a) Evaluate $\int \frac{x^2}{(a+bx)^3} dx$.

Or

(b) Evaluate $\int \frac{dx}{(1+e^x)(1+e^{-x})}$.

13. (a) Evaluate $I = \int_0^{\pi} \int_0^{a\cos\theta} \bar{r} \sin\theta \, dr \, d\theta$.

Or

(b) Evaluate $\int_0^2 \int_1^3 \int_1^2 xy^2 z \, dz \, dy \, dx$.

14. (a) If $\bar{f} = x^2 \bar{i} - xy \bar{j}$ and C is the straight line joining the points $(0, 0)$ and $(1, 1)$ find $\int_C f \cdot dr$.

Or

(b) Evaluate $\iint_S (x^2 + y^2) dS$ where S is the surface of the cone $z^2 = 3(x^2 + y^2)$ bounded by $z = 0$ and $z = 3$.

15. (a) Verify Gauss divergence theorem for the vector function $\bar{f} = (x^3 - yz)\bar{i} - 2x^2 y\bar{j} + 2\bar{k}$ over the cube bounded by $x = 0, y = 0, z = 0, x = a, y = a$ and $z = a$.

Or

(b) Verify Stokes theorem for the vector function $\bar{f} = y^2\bar{i} + y\bar{j} - xz\bar{k}$ and S is the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ and $z \geq 0$.

PART C — $(5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Find the equation of the

(i) tangent plane and

(ii) normal line to the surface $xyz = 4$ at the point $(1, 2, 2)$.

Or

(b) Prove that $curl(curl f) = grad\ div\ f - \nabla^2 f$.

17. (a) Evaluate $\int \frac{x}{\sqrt{x^2+x+1}}\,dx$.

Or

(b) Evaluate $\int \frac{dx}{(3+x)\sqrt{x}}$.

18. (a) Evaluate $I = \iint_D xy\,dy\,dx$ where D is the region bounded by the curve $x = y^2, x = 2-y, y = 0$ and $y = 1$.

Or

(b) Find by triple integral the volume of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$.

19. (a) If $\bar{f} = (2y+3)\,\bar{i} + xzj + (yz-x)\,\bar{k}$ evaluate $\int_C \bar{f}\cdot d\bar{r}$ along the following paths C

(i) $x = 2t^2; y = t; z = t^3$ from $t = 0$ to $t = 1$

(ii) The polygonal path P consisting of the three lines segments AB, BC, CD where $A=(0,0,0), B=(0,0,1), C=(0,1,1)$ and $D=(2,1,1)$.

(iii) The straight line joining $(0,0,0)$ and $(2,1,1)$.

Or

(b) Evaluate $\iint_S (\nabla \times \bar{f}) \cdot \bar{n}\, dS$ where $\bar{f} = y^2\,\bar{i} + y\,\bar{j} - xz\bar{k}$ and S is the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ and $z \geq 0$.

20. (a) Verify Gauss divergence theorem for the function $\bar{f} = a(x+y)\bar{i} + a(y-x)\bar{j} + z^2\bar{k}$ over the hemisphere bounded by the $x \circ y$ plane and the upper half of the sphere $x^2 + y^2 + z^2 = a^2$.

Or

(b) Using Green's theorem evaluate $\int_C (xy - x^2)dx + x^2 y dy$ along the closed curve C formed by $y=0, x=1$ and $y=x$.

(8 pages) **Reg. No. :** ....

Code No. : 40573 E **Sub. Code : SMMA 53**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Fifth Semester

Mathematics — Main

STATICS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The resultant of two forces $3P, 5P$ acting at an angle 60° is ————

(a) $7P$ (b) $8P$

(c) $2P$ (d) $\sqrt{7}P$

2. Converse of the polygon law of forces is ————

(a) true (b) not true

(c) some times true (d) none of these

3. Two parallel forces are said to be ———— when they act in the opposite direction.

(a) like (b) unlike

(c) equal (d) none of these

4. If O is any point and ON is the perpendicular from O on AB, then the moment of the force F about O is

(a) $2\,\Delta AOB$ (b) $2\,\Delta ONB$

(c) $2\,\Delta OAN$ (d) ΔAOB

5. If three forces acting on a rigid body are in equilibrium then they must be

(a) 0 (b) perpendicular

(c) coplanar (d) parallel

6. If there are only three non-parallel forces then they must ————

(a) meet at a point (b) perpendicular

(c) couple (d) none of these

7. A body of weight 4 kgs res in limiting equilibrium on an inclined plane whose inclination is 30°. Then the coefficient of friction is ________

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

8. The semivertical angle of the cone of friction is ________

(a) $\sin^{-1}(\mu)$ (b) $\tan^{-1}(\mu)$

(c) $\cos^{-1}(\mu)$ (d) 60°

9. The intrinsic equation of a common catenary is ________

(a) $s = c\tan\psi$ (b) $s = c\cos\psi$

(c) $s = c\sin\psi$ (d) $c = s\tan\psi$

10. In a parbolic catenary, tension $T =$ ________

(a) $w\sqrt{c^2 + 2y}$ (b) $w\sqrt{c^2 + 2cy}$

(c) $w\sqrt{c + y^2}$ (d) wcy

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) The resultant of forces P and Q is R. If Q is doubled, R is doubled, R also doubled if Q is a reversed, then show that $P : Q : R = \sqrt{2} : \sqrt{3} : \sqrt{2}$.

Or

(b) State and prove Lami's theorem.

12. (a) Derive the conditions of equilibrium of three coplanar parallel forces.

Or

(b) P and Q are two like parallel forces acting at points A and B respectively. If they interchange position, show that the point of application of the resultant will be displaced along AB through a distance $\frac{P - Q}{P + Q} \cdot AB$.

13. (a) If three forces acting on a rigid body are in equlibrium, show that they must be coplanar.

Or

(b) A heavy uniform sphere rests touching two smooth inclined planes one of which is inclined at 60° to the horizontal. If the pressure on this plane is one-half of the weight of the sphere, prove that the inclination of the other plane to the horizontal is 30°.

14. (a) State the laws of friction.

Or

(b) A uniform ladder AB rests in limiting equilibrium with the end A on a rough floor, the coefficient of friction being μ and with the other end B against a smooth vertical wall. Show that, if θ is the inclination of the ladder to the vertical, $\tan\theta = 2\mu$.

15. (a) Prove that the following common catenary equation.

(i) $y^2 = c^2 + s^2$

(ii) $x = c\log(\sec\psi + \tan\psi)$

Or

(b) A uniform chain of length l is suspended from two points A, B in the same horizontal line. If the tension at A is twice that at the lowest point, show that the span $AB = \frac{l}{\sqrt{3}}\log(2+\sqrt{3})$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions.

16. (a) State and prove converse of the triangle of forces.

Or

(b) P is a point in the plane of the triangle ABC and I is the incentre. Show that the resultant of the forces represented by $PA\sin A$, $PB\sin B$ and $PC\sin C$ is $4PI\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$.

17. (a) State and prove Varignon's theorem.

Or

(b) The resultant of three forces P, Q, R acting along the sides BC, CA, AB of a triangle ABC passes through the orthocentre. Show that the triangle must be obtuse angled. If $\angle A = 120^\circ$, and $B = C$, show that $Q + R = P\sqrt{3}$.

18. (a) Find the finite Fourier sine and cosine transform of $f(x) = e^{ax}$ in $(0, l)$.

Or

(b) Find the finite Fourier sine and cosine transform of $f(x) = x^3$, $0 < x < 4$.

19. (a) (i) If $f(K) = \left(\frac{1}{2}\right)^K$, then find $Z[f(K)]$

(ii) Find $Z[n^2]$.

Or

(b) State and prove the final value theorem for Z–transform.

20. (a) Find $Z^{-1}\left[\frac{z^2 + 2z}{z^2 + 2z + 4}\right]$.

Or

(b) Find $Z^{-1}\left[\frac{z^2}{z^2 + 4}\right]$ using residue theorem.

(8 pages) **Reg. No. :** ....

Code No. : 41157 E **Sub. Code : JMMA 52/ JMMC 52**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Fifth Semester

Mathematics/Mathematics with C.A. — Main

MECHANICS

(For those who joined in July 2016 Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If the resultant of two forces acting at a point is greatest, then the angle between them is

   (a) 180° (b) 90°

   (c) 0° (d) $\pi/4$

2. A force has no resolved part in a direction ———————— to itself.

   (a) parallel (b) perpendicular

   (c) 45° (d) 30°

3. The relation connectivity y and s is ————————.

   (a) $y^2 + x^2 = s^2$

   (b) $y^2 = c^2 + s^2$

   (c) $y^2 = c\cosh(x/c)$

   (d) $y^2 + s^2 = c^2$

4. The tension of any point on a common catenary is

   (a) ws (b) wc

   (c) wx (d) wy

5. The maximum horizontal range = ————————.

   (a) $\frac{g}{u^2}$ (b) $\frac{u^2}{g}$

   (c) $\frac{u}{2g}$ (d) $\frac{2u^2}{g}$

6. A projectile is thrown with a velocity of 20 m/sec at an elevation of 30°. The greatest height attained by the projectile is

   (a) 5.1 m (b) 5.5 m

   (c) 5.0 m (d) 5.3 m

7. In a SHM, the frequency of oscillation is ________.

(a) $\frac{\pi}{\sqrt{\mu}}$ (b) $\frac{2\pi}{\sqrt{\mu}}$

(c) $\frac{\sqrt{\mu}}{2\pi}$ (d) None

8. The displacement of S.H.M. is

(a) $x = a\cos\mu t$ (b) $x = -a\cos\sqrt{\mu}t$

(c) $x = a\cos\sqrt{\mu}t$ (d) $x = a^2\cos\sqrt{\mu}t$

9. The magnitude of the transverse component of acceleration is

(a) $\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})$ (b) $\frac{1}{r}\frac{d}{dt}(r\dot{\theta})$

(c) $\frac{d}{dt}(r\dot{\theta})$ (d) $\frac{1}{r^2}\frac{d}{dt}(r\dot{\theta})$

10. Differential equation of a central orbit is $u + \frac{d^2u}{d\theta^2} =$ ________.

(a) $\frac{p}{h^2}$ (b) $\frac{p}{h^2u^2}$

(c) $\frac{hu}{p}$ (d) $\frac{p}{hu}$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove the triangle law of forces.

Or

(b) Two like parallel forces P and Q act on a rigid body at A and B respectively. If Q be changed to $\frac{P^2}{Q}$, show that the line of action of the resultant is the same as it would be if the forces were simply interchanged.

12. (a) Derive the relation $x = c\log(\sec\psi + \tan\psi)$.

Or

(b) Let T be the tension at any point P of the string, T_0 is the tension at the lowest point C. Prove that $T^2 - T_0^2 = w^2s^2$, w being the weight of the arc CP and s = arc CP of the string.

13. (a) If the greatest height attained by the particle is a quarter of its range of the horizontal plane through the point of projection, find the angle of projection.

Or

(b) Find the range of a projectile on an inclined plane.

14. (a) A horizontal shelf moves vertically with S.H.M. whose complete period 1 second. Find the greatest amplitude in centemeters, it can have, so that an object resting on the shelf may always remain in contract.

Or

(b) Show that the composition of two simple harmonic motions of the same period in two perpendicularly directions is an ellipse.

15. (a) Derive the pedal equation of the central orbit.

Or

(b) Show that the a real velocity in a central orbit is constant.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Varigon's theorem of moments.

Or

(b) Forces P, Q, R act along the sides BC, CA, BA respectively of an equilateral triangle. If their resultant is a force parallel to BC through the centroid of the triangle, prove that $Q = R = \frac{P}{2}$.

17. (a) A uniform chain of length l is to be suspended from two points in the same horizontal line so that either terminal tension is n times than at the lowest point, show that the span must be $\frac{1}{\sqrt{n^2-1}} \log\left(n + \sqrt{n^2-1}\right)$.

Or

(b) Show that the length of an endless chain which will hang over a circular pulley of radius 'a' so as to be in contact with two-thirds of the circumference of the pulley is $a\left[\frac{3}{\log(2+\sqrt{3})} + \frac{4\pi}{3}\right]$.

18. (a) Show that the greatest height which a particle with initial velocity V can reach on a vertical wall at a distance 'a' from the point of projection is $\frac{V^2}{2g} - \frac{ga^2}{2V^2}$. Prove also that the greatest height above the point of projection attained by the particle in its flight is $\frac{V^6}{2g\left(V^4 + g^2a^2\right)}$.

Or

(b) A particle is projected at an angle α with a velocity u and it strikes up an inclined plane of inclination β at right angles to the plane. Prove that

(i) $\cot\beta = 2\tan(\alpha - \beta)$ and

(ii) $\cot\beta = \tan\alpha - 2\tan\beta$.

19. (a) Obtain the equation of SHM and solve completely.

Or

(b) A particle of mass m is oscillating in a straight line about a centre of force O, towards which when at a distance r, the force is mn^2r and 'a' is the amplitude of oscillation; when at a distance $\frac{a\sqrt{3}}{2}$ from O, a particle receives a below in the direction at motion which generates a velocity na. If the velocity be away from O, show that new amplitude is $a\sqrt{3}$.

20. (a) Derive the differential equation of a central orbit.

Or

(b) Find the law of force towards the pole under which the curve $r^n = a^n \cos n\theta$ can be described.

(7 pages) **Reg. No. :** ....

Code No. : 40569 E **Sub. Code : SMMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Second Semester

Mathematics — Main

ANALYTICAL GEOMETRY OF THREE DIMENSIONS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — $(10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. The direction cosines of the X-axis are ————.

   (a) (0, 1, 0) (b) (0, 0, 1)

   (c) (1, 0, 0) (d) (0, 0, 0).

2. Let a_1, b_1, c_1 and a_2, b_2, c_2 be the direction ratios of two lines. These two lines are parallel if ————.

   (a) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (b) $a_1a_2 + b_1b_2 + c_1c_2 = 0$

   (c) $\frac{a_1 + b_1 + c_1}{a_2 + b_2 + c_2} = 0$ (d) None.

3. If the lines $\frac{x-1}{1}=\frac{y-3}{4}=\frac{z+1}{5}$ and $\frac{x+1}{4}=\frac{y+1}{3}=\frac{z}{k}$ are perpendicular then $k =$ ————————.

(a) 3/5 (b) 7/5

(c) 5/8 (d) 8/5.

4. The point of intersection of the straight line $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{2}$ and the plane $2x+y+z-6=0$.

(a) (1, 2, 3) (b) (1, –2, 3)

(c) (1, 2, 2) (d) (2, 1, 2)

5. The radius of the sphere

$2x^2+2y^2+2z^2-2x+4y+2z-15=0$

is ————————.

(a) 2 (b) 3

(c) 15 (d) $\sqrt{2}$

6. In any equation of sphere the coefficient of xy is ————————.

(a) 2 (b) 0

(c) 1 (d) –2.

7. $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$ represents a cone if

(a) $\Delta=0$ (b) $\Delta\neq 0$

(c) $1/\Delta=0$ (d) None.

8. The axis of right circular cylinder $\frac{x+2}{3} = \frac{y-4}{6} = \frac{z-1}{2}$ passes through ————.

(a) $(2, -4, -1)$ (b) $(-2, 4, 1)$

(c) $(2, 4, 1)$ (d) None.

9. The locus of the centre of the parallel plane section of a conicoid is a ————.

(a) Radius (b) Diameter

(c) Vertex (d) None.

10. The intersection of $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ with the xy–Plane is ————.

(a) a parabola (b) an ellipse

(c) hyperbola (d) a circle.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that $(2, 5, -4)$, $(1, 4, -3)$, $(4, 7, -6)$ and $(5, 8, -7)$ are the vertices of a parallelogram.

Or

(b) Find the angle between two diagonals of a cube.

12. (a) Find the equation of the plane which passes through the points $(-1, 3, 2)$ and perpendicular to the planes $x+2y+2z=5$, $3x+3y+2z=8$.

Or

(b) Find the distance between the given parallel planes $2x-2y-z-3=0$ and $4x-4y+2z+5=0$.

13. (a) Find the equations of the image of the line $\frac{x-1}{2}=\frac{y+2}{-5}=\frac{z-3}{2}$ in the plane $2x-3y+2z+3=0$.

Or

(b) Find the condition for the lines

$$ax+by+cz+d=0=a_1x+b_1y+c_1z+d_1,$$

$$a_2x+b_2y+c_2z+d_2=a_3x+b_3y+c_3z+d_3$$

to be coplanar.

14. (a) A sphere of constant radius K passes through the origin and meets the axes in A, B, C. Prove that the centroid of the triangle ABC lies on the sphere $a\left(x^2+y^2+z^2\right)=4K^2$.

Or

(b) Find the equation of the sphere having the circle $x^2+y^2+z^2-2x+4y-6z+7=0$, $2x-y+2z=5$ as a great circle.

15. (a) Find the equation of the cone of the second degree which passes through the axes.

Or

(b) Find the equation of a right circular cylinder of radius 3 with axis $\frac{x+2}{3}=\frac{y-4}{6}=\frac{z-1}{2}$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) A line makes angle $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube then prove that

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}.$$

Or

(b) If the direction cosines of the two lines satisfy the equations $l+m+n=0$; $2\,lm+2\,ln-mn=0$; then find the angle between the lines.

17. (a) Find the equation of the plane passing through the points $(2, 5, -3)$, $(-2, -3, 5)$ and $(5, 3, -3)$.

Or

(b) Show that the origin lies in the acute angle between the plane $x + 2y + 2z = 0$, $4x - 3y + 12z + 13 = 0$. Find the plane bisecting the angles between them and point out which bisects the obtuse angle.

18. (a) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the axes in A, B, C. Find the coordinates of the orthocentre of the triangle ABC.

Or

(b) Find the shortest distance between the lines $\frac{x-3}{-3} = \frac{y-8}{1} = \frac{z-3}{-1}$, $\frac{x+3}{3} = \frac{y+7}{-2} = \frac{z-6}{-4}$ and find the equation of the line of shortest distance also.

19. (a) A plane passes through a fixed point (a, b, c) and cuts the axes in A, B, C. Show that the locus of the centre of the sphere $OABC$ is $a/x + b/y + c/z = 2$.

Or

(b) The plane ABC, whose equation is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the axes in A, B, C. Find the equation to the circumcircle of the ΔABC and obtain the coordinaets of its centre and radius.

20. (a) Find the condition for the equation

$$F(x, y, z) = ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz + d = 0$$

to represent a cone.

Or

(b) Derive the condition for the plane $lx + my + nz = 0$ to touch the quadric cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$.

———————

(7 pages) **Reg. No. :** ....

Code No. : 40574 E **Sub. Code : SMMA 54**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Fifth Semester

Mathematics – Core

TRANSFORMS AND THEIR APPLICATIONS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. $F[f(x-a)] =$ ————.

(a) $e^{ia}F(s)$ (b) e^{ias}

(c) $e^{ias}F(s)$ (d) $F(s)$

2. $F\left[\frac{1}{\sqrt{x}}\right] =$ ————.

(a) $\frac{1}{\sqrt{s}}$ (b) $\sqrt{s}$

(c) s (d) $\frac{1}{s}$

3. $F_c[f(x)] =$ ————.

(a) $\sqrt{\frac{\pi}{2}}\int_0^\infty f(x)\cos x\,dx$ (b) $\sqrt{\frac{2}{\pi}}\int_{-\infty}^\infty f(x)\cos x\,dx$

(c) $\sqrt{\frac{\pi}{2}}\int_{-\infty}^\infty f(x)\cos x\,dx$ (d) $\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos x\,dx$

4. $F_s[e^{-ax}] =$ ————.

(a) $\sqrt{\frac{2}{\pi}}\left(\frac{s}{s^2+a^2}\right)$ (b) $\sqrt{\frac{2}{\pi}}\left(\frac{s}{s^2-a^2}\right)$

(c) $\sqrt{\frac{\pi}{2}}\left(\frac{s}{s^2+a^2}\right)$ (d) $\sqrt{\frac{\pi}{2}}\left(\frac{s}{s^2-a^2}\right)$

5. $F_s[x]$ in $(0,\pi)$ is ————.

(a) $(-1)^n \frac{\pi}{n}$

(b) $(-1)^n \frac{\pi}{n+1}$

(c) $(-1)^{n+1} \frac{\pi}{n!}$

(d) $(-1)^{n+1} \frac{\pi}{n}$

6. $F_c[f(x)]$ in $(0, l)$ is ————.

(a) $\int_0^l f(x) \cos n\pi x \, dx$

(b) $\int_0^l f(x) \frac{\cos n\pi x}{l} dx$

(c) $\frac{2}{\pi} \int_0^l f(x) \frac{\cos n\pi x}{l} dx$

(d) $\frac{2}{\pi} \int_0^l f(x) \cos n\pi x \, dx$

7. $Z[(-1)^n] =$ ————.

(a) $\frac{z}{z+1}$

(b) $\frac{z}{z-1}$

(c) $\frac{1}{z+1}$

(d) $\frac{1}{z-1}$

8. $Z\left[\cos\frac{n\pi}{2}\right] =$ ————.

(a) $\frac{z}{z+1}$

(b) $\frac{z^2}{z^2+1}$

(c) $\frac{z^2}{z^2-1}$

(d) $\frac{z}{z^2-1}$

9. $Z^{-1}\left[\frac{az}{(z-a)^2}\right]$ is ————.

(a) $\frac{a^n}{n}$

(b) $\frac{a^n}{n+1}$

(c) na^n

(d) na^{n+1}

10. $Z^{-1}\left[e^{\frac{1}{z}}\right] =$ ————.

(a) $\frac{1}{n}$

(b) $n!$

(c) $\frac{1}{n!}$

(d) n

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find $F\left[e^{-x^2/2}\right]$.

Or

(b) State and prove the convolution theorem for Fourier transform.

12. (a) Find the Fourier cosine transform of $\left[\frac{e^{-ax}-e^{-bx}}{x}\right]$.

Or

(b) Find $F_s\left[\frac{x}{x^2+a^2}\right]$.

13. (a) Find finite Fourier cosine transform of $f(x)=\frac{\pi}{3}-x+\frac{x^2}{2\pi}$ in $(0,\pi)$.

Or

(b) Find the finite sine transform of $f(x)=\cos Kx$ in $0<x<\pi$.

14. (a) Prove that $Z\left[\frac{1}{n+1}\right]=z\log\left(\frac{z}{z-1}\right)$.

Or

(b) If $Z[f(t)]=F(z)$, then prove that $\lim_{t\to\infty} f(t)=\lim_{z\to 1}(z-1)F(z)$.

15. (a) Find $Z^{-1}\left[\frac{z^2-3z}{(z-5)(z+2)}\right]$.

Or

(b) Find $Z^{-1}\left[\frac{1}{(1+z^{-1})(1-z^{-1})^2}\right]$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Find the Fourier transform of $f(x)$ if $f(x)=\begin{cases}1-|x|, & if\ |x|<1\\ 0, & if\ |x|>1\end{cases}$. Hence deduce that $\int_0^\infty\left(\frac{\sin t}{t}\right)^4 dt-\frac{\pi}{3}$.

Or

(b) Prove that $F[x^n f(x)]=(-i)^n\frac{d^n F(s)}{ds^n}$.

17. (a) Find $F_s[x^{n-1}]$ and $F_c[x^{n-1}]$, $0<n<1$. Hence show that $\frac{1}{\sqrt{x}}$ is self-reciprocal under both the transforms.

Or

(b) Using Parseval's identity, calculate

(i) $\int_0^\infty\frac{dx}{(a^2+x^2)^2}$ and

(ii) $\int_0^\infty\frac{x^2}{(a^2+x^2)^2}dx$ if $a>0$.

20. (a) (i) Convert the binary number 1011.01101 into an octal number

(ii) Find the binary equivalent of the decimal 363.

Or

(b) (i) Find the octal expansion of $(12345)_{10}$

(ii) Find the binary expansion of $(2\ 4\ 1)_{10}$.

(8 pages) **Reg. No. :** ....

Code No. : 41179 E **Sub. Code : JMMA 5 C**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Fifth Semester

Mathematics — Main

Major Elective I – DISCRETE MATHEMATICS

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. $P \vee (P \wedge Q)$ is equivalent to ————

(a) Q (b) P

(c) $P \vee Q$ (d) $P \wedge Q$

2. $\urcorner(P \vee Q) \Leftrightarrow$ ————

(a) $\urcorner(P \vee Q)$ (b) $(P \wedge Q)$

(c) $\urcorner P \wedge \urcorner Q$ (d) $\urcorner P \vee \urcorner Q$

3. For any commutative monoid $< m, * >$, the set of idom potend element of M forms a ————

(a) Semi group (b) Idempotent

(c) Monoid (d) Submonoid

4. $a^H = \{a * h \ (h \in H\}$ is ————

(a) coset

(b) left coset of H in G

(c) right coset of H in G

(d) None

5. An algebra $(s, *, \oplus)$ is ———— of $(L ; *, \oplus)$ iff S' is closed under both operations $*$ and $\oplus$

(a) lattice (b) sub lattice

(c) group (d) monoid

6. $a \oplus a = a$ is called ————

(a) absorption (b) associative

(c) commutative (d) idempotent

7. $g(S_i . x_0, x_1) =$ ______________

(a) $\delta(\delta(S_i, x_0), x_1)$

(b) $\lambda(\delta(S_i, x_0), x_1)$

(c) $\lambda(S_i, x_0)\lambda(S_i, x_1)$

(d) none

8. If $a \le b$ then $G\ L\ B\ \{a, b\}$

(a) a (b) b

(c) 0 (d) $a \oplus b$

9. Let a and m be relatively prime then for any a ______________

(a) $a^m = 1$ (b) $a^{f(m)} \bmod m = 1$

(c) positively integer m (d) none

10. For $0 \le a < m$ if there exist an a' such that (a', a) modem $= 1$ then a' is the ______________ of a

(a) additionally inverse

(b) subtractionally inverse

(c) divisionally inverse

(d) multiplicative inverse

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) (i) Show that

$$\neg P\,(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q).$$

(ii) $$(P \vee Q) \wedge (\neg P \wedge (\neg P \wedge Q)) \Leftrightarrow (\neg P \vee Q).$$

Or

(b) (i) Show that

$$((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$$

is a tautology.

(ii) Define well formed formula.

12. (a) Let $<m, *>$ be a monoid. Then prove that there exist subset $T \in M$ such that $<m, *>$ is isomorphic to the monoid $<T, O>$.

Or

(b) Show that the semigroup $< X, * >$ in which $X = \{a, b, p, q\}$ and the operation $*$ is given by generated by the set $\{a, b\}$.

13. (a) Define Boolean algebra write down its properties

Or

(b) Obtain the sum of product of Boolean expression.

(i) $n_1 * n_1$

(ii) $n_1 \oplus n_2$

(iii) $(n_2 \oplus n_2)' * (n_3)$.

14. (a) Let $S_i\ S_j \in \delta$ then prove that then $S_i \underline{\underline{k+1}}\ S_j$ iff $S_i \underline{\underline{k}}\ S_j$ and for all $a \in S$, $\delta\ (S_i, a) \underline{\underline{k}}\ \delta\ (S_j, a)$.

Or

(b) Define transition diagram.

15. (a) State and prove Euler's theorem.

Or

(b) Prove that the quantity a' exist and is unique iff $G \subset D\ (a, m) = 1$ and $a \neq 0$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Obtain the principal disjunctive normal form of $P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$.

Or

(b) Obtain the conjuctive normal form of

(i) $P \wedge (P \rightarrow Q)$

(ii) $\neg(P \vee Q) \rightleftarrows (P \wedge Q)$

17. (a) Let $< S, * >$ and $< T, \Delta >$ be two semi groups and g be a semi groups homomorphism from $< S, * >$ to $< T, \Delta >$. Show that corresponding to the homomorphism. For there exists a congruence relation R on $< S, * >$ defined by, $n\,R\,y$ iff $g\ (x) = g\ (y)$ for $x, y \in S$.

Or

(b) Let $M = \{1, 2 \ldots m\}$ and τ be unary operation on M, prove that

$$\tau(j) = \begin{cases} j+1 & j \neq m \\ 1 & j = m \end{cases}.$$

18. (a) Show that every chain is distributive lattice.

Or

(b) Minimize the Boolean function $f(a, b, c, d) = \Sigma(3, 4, 5, 7, 9, 13, 14, 15)$ using Karnaugh map.

19. (a) Computing the output function g, given an input sequence $\| 0 \|$.

Or

(b) Let $M = (I, S, O, \delta. \lambda)$ be a finite state machine then show that their an equivalent machine m with a set of states such that $S' \subset S$ and m' reduced.

20. (a) State and prove Euler's theorem.

Or

(b) Let H be matrix which consist of k rows and n columns then prove that the set of words $x = < x_1, x_2 \ldots x_n >$ which belongs to the following set $C = \{x \mid (x \,.\, H^t = 0) \bmod 2\}$ is a group. Code under the operation $\oplus$.

(6 pages) **Reg. No. :** ....

Code No. : 40354 E **Sub. Code : JNMA 3 A/ JNMC 3 A/SNMA 3 A**

U.G. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Third Semester

Mathematics/ Mathematics with CA — Non-Major-Elective

MATHEMATICS FOR COMPETITIVE EXAMINATIONS – I

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. $\frac{11}{4} = \frac{77}{?}$

(a) 28 (b) $\frac{77}{28}$

(c) 44 (d) 308.

2. The mean of $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2$ is ________

(a) 10 (b) 20

(c) 30 (d) 40.

3. If $a:b=5:9$ and $b:c=4:7$, then $a:b:c=$ ________.

(a) 1 : 2 : 3 (b) 3 : 35 : 41

(c) 20 : 36 : 63 (d) 25 : 36 : 63.

4. The fourth proportional to 4, 9, 12 is ________

(a) 30 (b) 29

(c) 31 (d) 27.

5. *A, B, C* started a business by investing Rs. 1,20,000, Rs. 1,35,000 and Rs. 1,50,000 respectively. The *A*'s share out of an annual profit of Rs. 56,700 ________.

(a) Rs. 15,000 (b) Rs. 16,800

(c) Rs. 17,800 (d) Rs. 20,000.

6. The decimal of 6% is ________.

(a) 0.06 (b) 60.0

(c) 6 (d) 0.6.

7. When SP = Rs. 40.60 and Gain = 16%, CP = ____.

(a) Rs. 35 (b) Rs. 30

(c) Rs. 45 (d) Rs. 40.

8. If loss is 1/3 of SP, then the loss percentage is ________

(a) $16\frac{2}{3}$!
(b) 20%
(c) 25%
(d) $33\frac{1}{3}$!.

9. The difference between a number and its three fifth is 50. What is the number?

(a) 75
(b) 100
(c) 125
(d) None of these.

10. The number is double and 9 is added. If the resultant is tripled, it becomes 75. Then that number is ________

(a) 3.5
(b) 6
(c) 8
(d) 12.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the value of $4 - \cfrac{5}{1 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{4}}}}$.

Or

(b) The average of four consecutive even number is 27. Find the largest of these number.

12. (a) If $x : y = 3 : 4$ then find $(4x + 5y) : (5x - 2y)$.

Or

(b) Two numbers are respectively 20% and 50% more than a third number. Find the ratio of the two numbers.

13. (a) *A, B, C* enter into a partnership investing Rs. 35,000, Rs. 45,000 and Rs. 50,000 respectively. Find the respective shares of *A, B, C* in an annual profit of Rs. 40,500?

Or

(b) If the sales tax be reduced from $3\frac{1}{2}\%$ to $3\frac{1}{3}\%$ then what difference does it make to a person who purchases an article with marked price of Rs. 8,400?

14. (a) A book was sold for Rs. 27.50 with a profit of 10%. If it were sold for Rs. 25.75, then what would have been the percent of profit or loss?

Or

(b) If loss is 1/3 of SP, find the loss percentage.

15. (a) 50 is divided into two parts such that sum of their reciprocals is 1/12. Find the two parts.

Or

(b) The product of two numbers is 120 and the sum of their square is 289. Then find the sum of the numbers.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Find the value of x if

$$8.5-\left\{5\tfrac{1}{2}-\left(7\tfrac{1}{2}+2.8\div x\right)\right\}\times 4.25\div(0.2)^2=306.$$

Or

(b) The average age of a class of 39 students is 15 years. If the age of the teacher be included, then the average increases by 3 months. Find the age of the teacher.

17. (a) A bag contains 50 P, 25 P and 10 P coins in the ratio 5 : 9 : 4 amounting to Rs. 206. Find the number of coins of each type.

Or

(b) A sum of Rs. 1,300 is divided amongst P, Q, R and S such that

$$\frac{P\text{'s share}}{Q\text{'s share}}=\frac{Q\text{'s share}}{R\text{'s share}}=\frac{R\text{'s share}}{S\text{'s share}}=\frac{2}{3}.$$

18. (a) Three persons A, B, C are joined as shareholders in the ratio 3 : 2 : 4. After one year, the person B raised his share amount to Rs. 2,70,000 and after two years, the person C raised his share amount to Rs. 2,70,000. If their share ratio is 3 : 4 : 5 at the end of three years, find their initial share amount.

Or

(b) In an examination, 80% of the students passed in English, 85% in Mathematics and 75% in both English and Mathematics. If 40 students failed in both the subjects, find the total number of students.

19. (a) A dealer sold three-fourth of his articles at a gain of 20% and the remaining at cost price. Find the gain earned by him in the whole transaction.

Or

(b) A vendor loses the selling price of 4 oranges on selling 36 oranges. Find his loss percentage.

20. (a) The sum of a national number and its reciprocal is 13/6. Find the number.

Or

(b) The product of two natural numbers is 17. Find the sum of the reciprocal of their squares.

(7 pages) **Reg. No. :** ....

Code No. : 40576 E **Sub. Code : SEMA 5 B**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Fifth Semester

Mathematics — Core

Major Elective — DISCRETE MATHEMATICS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Complete the table:

P	Q	$P \nabla Q$
T	T	?

(a) T (b) F

(c) T or F (d) T and F

2. If P has truth value T, the truth value of $P \wedge \neg P$ is ————

(a) T (b) F

(c) T and F (d) T or F

3. $R \vee (P \wedge \neg P) \Leftrightarrow$

(a) P (b) R

(c) $\neg$ P (d) None of these

4. The PDNF for $\neg P \vee Q$ is ————

(a) $(P \wedge Q) \vee (\neg P \wedge Q) \vee (\neg P \wedge \neg Q)$

(b) $(P \vee Q) \wedge (\neg P \vee Q)$

(c) $(P \wedge \neg Q) \vee (\neg P \wedge Q)$

(d) None of these

5. In $(\mathbb{N}, *)$, * is defined as $x * y = \max\{x, y\}$. Then $\mathbb{N}$ is a ————

(a) Group

(b) Ring

(c) Monoid

(d) Semigroup and monoid

6. $(a * b) * c =$ ————

(a) $a * c$ (b) $b * c$

(c) $a * b$ (d) $a * (b * c)$

7. **In a Lattice with usual notations,** the value of $a \oplus (a * b)$ is ________

(a) a (b) $a * b$

(c) $b * b$ (d) $a \oplus b$

8. A ________ distributive lattice is called a Boolean Algebra

(a) Complemented (b) Modular

(c) Universal (d) None of these

9. The binary number for decimal 17 is ________

(a) 10011 (b) 11011

(c) 10001 (d) 11001

10. A binary number has 9 bits. The binary weight of the most significant bit is ________

(a) 215 (b) 512

(c) 152 (d) 251

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Construct the truth table for the formula $((P \to Q) \wedge (Q \to R)) \to (P \to R)$.

Or

(b) Indicate whether the formula $(P \wedge Q) \wedge \neg (P \vee Q)$ is a tautology or contradication.

12. (a) Find whether the conclusion C follows from the premises H_1, H_2, H_3 in the following cases, using truth table technique: $H_1 : \neg P, H_2 : P \vee Q, C : P \wedge Q$.

Or

(b) Without using truth table, $(\neg p \to q) \wedge (q \leftrightarrow p)$ find PNDF.

13. (a) Let $|W| = \{0, 1, 2, ...\}$ and $S = \{e, 0, 1\}$ such that $(|W|, +)$ and $(S, *)$ are monoids. Amapping $g : W \to S$ is define by $g(o) = 1$ $g(j) = 0$ for $j \neq 0$, verify that g is a monoid homomorphism or not. If not, explain.

Or

(b) Let $(G, *)$ be a group such that $(a * b)^2 = a^2 * b^2$, $\forall a, b \in G$. Prove that $(G, *)$ is abelian.

14. (a) In a lattice if $a \leq b \leq c$, show that

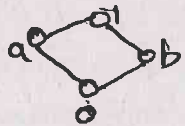
(i) $a \oplus b = b * c$

(ii) $(a * b) \oplus (b * c) = (a \oplus b) * (a \oplus c) = b$.

Or

(b) Find the value of $x_1 * x_2 * [(x_1 * x_2) \oplus x_2^1 \oplus (x_3 * x_1^1)]$

for $x_1 = a_1\ x_2 = 1,\ x_3 = b,\ x_4 = 0$ where $a_1\ b \in B$ the Boolean algebra $\langle B, *, \oplus, 0, 1\rangle$

is as. Shown below 

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15. (a) Convert decimal 23.6 to a binary number.

Or

(b) Convert the decimal 175 to octal number and covert the octal number 257 to decimal number.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Using truth table, show that $(P \to Q) \wedge (Q \to R) \to (P \to R)$ is a tautology.

Or

(b) Write a note on connectives.

17. (a) Obtain PDNF and PCNF of $(P \wedge Q) \vee (\neg P \vee R) \vee (Q \wedge R)$.

Or

(b) Find whether the conclusion C follows from the premises H_1, H_2, H_3 in the following cases, Using truth table technique :
$H_1 : p \vee q,\ H_2 : p \to r,\ H_3 : q \to r,\ c : r$.

18. (a) State and prove Lagrange's theorem for finite groups.

Or

(b) If $(G, *)$ is a group, prove that $(a * b)^{-1} = b^{-1} * a^{-1}$.

19. (a) Find the sum-of-product form of the Boolean function $f(x, y, z, w) = xy + y\overline{w}z$.

Or

(b) In any Boolean algebra, show that

$$(a + b')(b + c')(c + a') = (a' + b)(b' + c)(c' + a).$$

(8 pages) **Reg. No. :** ....

Code No. : 41184 E **Sub. Code : JMMA 5 E/ JMMC 5 E**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Fifth Semester

Mathematics/Mathematics with CA – Main

Major Elective — OPERATIONS RESEARCH

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Which of the following is not related to simplex method?

   (a) Feasible solution (b) Basic solution

   (c) Least cost (d) Optimal solution

2. If a primal variable is positive then the corresponding dual constraint is ———— of the optimum

   (a) an equation (b) ≤

   (c) ≥ (d) None

3. A method of testing optimality of a transportation problem is ————

   (a) N.W.C rule (b) Least cost method

   (c) VAM method (d) Modi method

4. Which of the following is a method of solving the assignment problem?

   (a) MODI (b) Matrix minima

   (c) Hungarian (d) VAM

5. Saddle point is the point of intersection of ———— pure strategies

   (a) three (b) four

   (c) two (d) five

6. The value of the game $\begin{bmatrix} 3 & -2 \\ -2 & 5 \end{bmatrix}$ is

   (a) $\frac{11}{8}$ (b) $\frac{-4}{3}$

   (c) $\frac{1}{12}$ (d) $\frac{11}{12}$

7. An activity which must be completed before one or more other activities start is ———— activity

   (a) predecessor (b) successor

   (c) dummy (d) none

8. An activity in a network diagram is said to be ————

   (a) critical (b) non-critical

   (c) initial (d) none

9. Which one of the following cost is a 4 cost of EOQ?

   (a) Investment cost (b) Profit cost

   (c) Set up cost (d) None

10. The time gap between the placement of an order and its actual arrival is known as ————

(a) Administrative time

(b) Delivery time

(c) Lead time

(d) None of the above

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Write down the simplex algorithm.

Or

(b) Prove that, the dual of the dual is the primal.

12. (a) Determine on initial basic feasible solution using matrix minima.

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirements	200	225	275	250	

Or

(b) Solve the following assignment problem

	1	2	3	4
A	10	12	19	11
B	5	10	7	8
C	12	14	13	11
D	8	15	11	9

13. (a) Check whether the game is strictly determinable and fair

$$A \overset{B}{\begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}}$$

Or

(b) Solve the following game

$$A \overset{B}{\begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}}$$

14. (a) Write down the rules of network construction.

Or

(b) For the activities $B,C,...Q$ and N the following relation is maintained: $B < E,F; C < G,L; E,G < H;$ $L,H < I;$ $L < M;$ $H,M < N;$ $H < J;$ $I,J < P; P < Q$. Construct a network.

15. (a) Write down the characteristics of fundamental EOQ problem.

Or

(b) Solve the production EOQ problem with shortages.

SECTION C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve the L.P.P by the simplex method

Max $Z = x_1 - x_2 - 3x_3$

Sub to constraints

$x_1 + x_2 + x_3 \le 10$

$2x_1 - x_3 \le 2$

$2x_1 - 2x_2 + 3x_3 \le 0$

$x_1, x_2, x_3 \ge 0.$

Or

(b) Use dual simplex method and solve,

Min $Z = 10x_1 + 6x_2 + 2x_3$

Sub to constraints :

$-x_1 + x_2 + x_3 \ge 1$

$3x_1 + x_2 - x_3 \ge 2.$

17. (a) Solve the following transportation problem

	D_1	D_2	D_3	D_4	
O_1	5	3	6	2	19
O_2	4	7	9	1	37
O_3	3	4	7	5	34
	16	18	31	25	

Or

(b) Solve the assignment problem.

	I	II	III	IV	V
1	3	8	2	10	3
2	8	7	2	9	7
3	6	4	2	7	5
4	8	4	2	3	5
5	9	10	6	9	10

18. (a) Solve the following problem graphically.

Player B

Player A	3	–3	4
	–1	1	–3

Or

(b) Is the following two person zero-sum game stable? Solve the game:

Player B

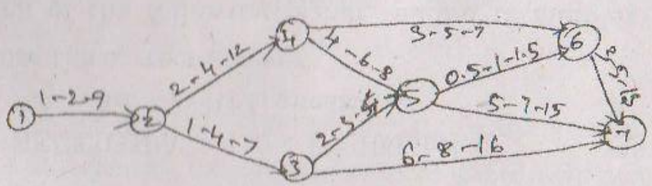
Player B	5	–10	9	0
	6	7	8	1
	8	7	15	2
	3	4	–1	4

19. (a) Tasks $A, B, \ldots H, I$ constitute $A < D$, $A < E$, $B < F$, $D < F$, $C < G$, $C < H$, $F < I$, $G < I$. Draw a graph to represent the sequence of tasks and find the minimum time of completion of the project, when the time of completion of each task is follows :

Task:	A	B	C	D	E	F	G	H	I
Time :	8	10	8	10	16	17	18	14	9

Or

(b) Consider the network shown in below :



Find the probability of completing the project in 25 days.

20. (a) A manufacturing company purchases 9,000 parts of a machine for its annual requirements, ordering one month usage at a time, Each part cost Rs. 20. The ordering cost per order is Rs. 15 and the carrying charges are 15% of the average inventory per year. You have been assigned to suggest a more economical purchasing policy for the company. What advice would you offer and how much would it save the company per year?

Or

(b) A company producing three items has a limited storage space of average 750 items of all types. Determine the optimal production quatities for each item separately when its following information is given.

Product	1	2	3
Holding cost ₹	0.05	0.02	0.04
Set up cost ₹	50	40	60
Demand (per unit)	100	120	75

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Code No. : 40355 E Sub. Code : JNMA 3 B/ JNMC 3 B/SNMA 3 B

U.G. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Third Semester

Mathematics/ Mathematics with CA — Non-Major – Elective

FUNDAMENTALS OF STATISTICS — I

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Which is the two-dimensional diagram?

   (a) sphere

   (b) cylinder

   (c) rectangle

   (d) cube.

2. A simple formula to obtain the estimate of appropriate class interval $i =$

(a) $L + S$

(b) $\frac{L - S}{k}$

(c) $\frac{L + S}{k}$

(d) $L - S$.

3. The geometric mean of numbers 2, 4, 8 is

(a) 1

(b) 2

(c) 3

(d) 4.

4. The mode of grouped frequency distribution is

(a) $l + \frac{\left(\frac{N}{2} - m\right) h}{f_k}$

(b) $l + \frac{hf_2}{f_1 + f_2}$

(c) $l - \frac{hf_2}{f_1 + f_2}$

(d) $l + \frac{hf_2}{f_1 - f_2}$.

5. Which is correct?

(a) Q.D. $= \frac{1}{2}(Q_3 - Q_1)$

(b) S.D. $\left[\frac{1}{N} \varepsilon \, f_i \left(x_i + \bar{x}\right)^2\right]^{1/2}$

(c) $\sigma^2 = s^2 + d^2$ where $d = \bar{x} - A$.

(d) C.V. $= \frac{\bar{x}}{\sigma} \times 100$.

6. The median of the numbers 2, 5, 4, 11, 8

(a) 2 (b) 4

(c) 5 (d) 8.

7. The range of the correlation coefficient is

(a) −1, 0 (b) 0, 1

(c) −1, 1 (d) 1, α.

8. The formula for finding the rank correlation

(a) $1-\frac{6\Sigma(x-y)^2}{n(n^2-1)}$ (b) $1+\frac{6\Sigma(x-y)^2}{n(n^2-1)}$

(c) $1-\frac{6\Sigma(x+y)^2}{n(n^2-1)}$ (d) $1-\frac{6\Sigma(x-y)^2}{n(n^2+1)}$.

9. The regression coefficient $b_{xy} =$

(a) $r\frac{\sigma_y}{\sigma_x}$ (b) $r\frac{\sigma_x}{\sigma_y}$

(c) $\frac{1}{r}\frac{\sigma_x}{\sigma_y}$ (d) $\frac{1}{r}\frac{\sigma_y}{\sigma_x}$.

10. If the regression coefficient of x on y is 0.4 and the regression coefficient of y on x is 0.9 then the correlation coefficient between x and y is

(a) 0.06 (b) 0.36

(c) 0.6 (d) 0.036.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Write down the objects of classification.

Or

(b) The dividend given by Oswal Agro Mills Ltd. from 1983 to 1988 is given below :

Year :	1983	1984	1985	1986	1987	1988
Dividend (%) :	20	30	32	42	50	50

Represent the data by bar diagram.

12. (a) From the following data, compute the value of harmonic mean.

Marks :	10	20	25	40	50
No. of Students :	20	30	50	15	5

Or

(b) The mean weight of 150 students in a certain class is 60 kg. The mean weight of boys in the class is 70 kg and that of girls in the class is 55 kg. Find the number of boys and girls in the class.

13. (a) Find (i) range (ii) standard deviation for the following marks of 10 students.

20, 22, 27, 30, 40, 48, 45, 32, 31, 35.

Or

(b) Show that the variance of the first n natural numbers is $\frac{(n^2-1)}{12}$.

14. (a) Calculate Karl Pearsons coefficient of correlation from the following data :

Marks in Mathematics :	48	35	17	23	47
Marks in Statistics :	45	20	40	25	45

Or

(b) Obtain the rank correlation coefficient for the following data :

x :	2	1	4	3	5	7	6
y :	1	3	2	4	5	6	7

15. (a) From the given equation to the two regression lines $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$, find the correlation coefficient between x and y.

Or

(b) Construct the two regression lines from the following data :

	x	y
Mean	10	90
Standard deviation	3	12

Correlation coefficient 0.8.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Explain the types of Bar diagram.

Or

(b) Draw a Pie diagram for the following data of sixth five year plan public sector outlays.

Agriculture and Rural development	12.9%
Irrigation etc	12.5%
Energy	27.2%
Industry and minerals	15.4%
Transport and Communication etc	15.9%
Social services and others	16.1%

17. (a) Find the mean and median for the following frequency distribution :

Class :	11 – 15	16 – 20	21 – 25	26 – 30	31 – 35
Frequency :	8	15	39	47	52
Class :	36 – 40	41 – 45	46 – 50	51 – 55	
Frequency :	41	28	16	4	

Or

(b) Given that the mode of the following frequency distribution of the 70 students is 58.75. Find the missing frequencies f_1 and f_2.

Class :	52 – 55	55 – 58	58 – 61	61 – 64
Frequency :	15	f_1	25	f_2

18. (a) The following table gives the marks obtained by a group of 80 students in an examination. Calculate the variance.

Marks obtained :	10 – 14	14 – 18	18 – 22	22 – 26	26 – 30	30 – 34
No. of students :	2	4	4	8	12	16
Marks obtained :	34 – 38	38 – 42	42 – 46	46 – 50	50 – 54	54 – 56
No. of students :	10	8	4	6	2	4

Or

(b) Compute Quartile Deviation from the following data :

x :	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
y :	12	19	5	10	9	6

19. (a) Calculate the correlation coefficient for the following heights in inches of fathers (X) and their sons (Y).

X :	65	66	67	67	68	69	70	72
Y :	67	68	65	68	72	72	69	71

Or

(b) Find the rank correlation coefficient for the following data :

x :	68	64	75	50	64	80	75	40	55	64
y :	62	58	68	45	81	60	68	48	50	70

20. (a) (i) Write short notes on regression lines.

(ii) Out of the two lines of regression given by $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$ which one is the regression line of x on y.

Or

(b) Obtain the two regression equations from the following data :

x :	6	2	10	4	8
y :	9	11	5	8	7

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Code No. : 40578 E **Sub. Code : SEMA 5 D**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Fifth Semester

Mathematics — Core

Major Elective — OPERATIONS RESEARCH — I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. In LPP the objective function subject to a set of linear equation (or) inequalities is known as

   (a) Constraints

   (b) Equations

   (c) Objective function

   (d) None

2. The leading element obtained in simplex table is also called

(a) Pivotal element

(b) Minimum element

(c) Bonded element

(d) Unbounded element

3. In a LPP the number of variables is 3 and the number of constraints is 2, then the constraints of the dual is ————.

(a) 2

(b) 3

(c) 6

(d) 4

4. The dual of the dual is

(a) Dual

(b) Primal

(c) Optimum

(d) Unbounded

5. A transportation problem is balanced if ————.

(a) Total supply > Total demand

(b) Total supply = 0

(c) Total supply = Total demand

(d) Total demand = 0

(b) Solve the following assignment problem :

	E	F	G	H
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

20. (a) Solve the following sequence problem

Job	A	B	C	D	E	F	G
Machine M_1	3	8	7	4	9	8	7
Machine M_2	4	3	2	5	1	4	3
Machine M_3	6	7	5	11	5	6	12

Or

(b) Use the graphical method solve the following 2 jobs 5 machines sequencing problem :

Job I	Sequence (Time in hrs)	A	B	C	D
		2	3	5	2
Job II	Sequence (Time in hrs)	D	C	A	B
		6	2	3	1

6. ________ method is used to find the initial basic feasible solution of a transportation problem.

(a) VAM (b) MODI

(c) Euler (d) Horney

7. The method of solving an assignment problem is

(a) Modi method (b) Hungarian method

(c) Vogel's method (d) Two-phase method

8. Assignment model is a special case of ________.

(a) Transportation (b) Sequencing

(c) Routing (d) None of these

9. The time for which the machine has no job to process is ________ on machine.

(a) Total time (b) Processing time

(c) Idle time (d) None

10. Sequencing problems involving processing of two jobs on 'n' machines

(a) can be solved by graphical method

(b) cannot be solved by graphical method

(c) have a condition that the processing of two jobs must be in the same order

(d) none of these

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve graphically the following LPP

Maximize $z = 4x_1 + 3x_2$

Subject to

$2x_1 - 3x_2 \le 6$

$6x_1 + 5x_2 \ge 30$

$x_1, x_2 \ge 0$.

Or

(b) Write down the standard form of the following LPP :

Minimize $z = 2x_1 + 5x_2 + x_3$

Subject to

$x_1 + 3x_2 - 4x_3 \le 20$

$2x_1 + x_2 + x_3 \ge 10$

$x_1 + 4x_2 + 5x_3 = 10$

$x_1, x_2, x_3 \ge 0$.

12. (a) Write down the dual of :

Maximize $z = 3x_1 + 10x_2 + 2x_3$

Subject to

$2x_1 + 3x_2 + 2x_3 \le 7$

$3x_1 - 2x_2 + 4x_3 = 3$

$x_1, x_2, x_3 \ge 0$.

Or

18. (a) Solve the following transportation problem :

	$W1$	$W2$	$W3$	a_i
$F1$	8	10	12	900
$F2$	12	13	12	1000
$F3$	14	10	11	1200
b_j	1200	1000	900	3100

Or

(b) Solve the following transportation problem :

	D_1	D_2	D_3	D_4	a_i
S_1	3	1	7	4	300
S_2	2	6	5	9	400
S_3	8	3	3	2	500
b_j	250	350	400	200	1200

19. (a) Solve the following assignment problem :

	M_1	M_2	M_3
J_1	19	28	31
J_2	11	17	16
J_3	12	15	13

Or

(b) Solve the following by big M-method :

Maximize $z = x_1 + 2x_2$

Subject to

$2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0.$

17. (a) Solve by simplex method using dual of the following LPP

Minimize $z = 2x_1 + 3x_2$

Subject to

$x_1 + x_2 \geq 5$

$x_1 + 2x_2 \geq 6$

$x_1, x_2 \geq 0.$

Or

(b) Use dual simplex method to solve :

Minimize $z = 4x_1 + x_2$

Subject to

$3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0.$

(b) Use dual simplex method to solve :

Maximize $z = 2x_1 + 3x_2$

Subject to

$2x_1 - x_2 - x_3 \geq 3$

$x_1 - x_2 + x_3 \geq 2$

$x_1, x_2, x_3 \geq 0.$

13. (a) Using North-west corner rule find an initial basic feasible solution for the following transportation problem :

	$W1$	$W2$	$W3$	a_i
$F1$	2	7	4	5
$F2$	3	3	1	8
$F3$	5	4	7	7
$F4$	1	6	2	14
b_j	2	9	18	29 \ 34

Or

(b) Find the initial basic feasible solution for the following transportation problem using VAM method.

	D_1	D_2	D_3	D_4	
a_1	2	3	11	7	8
a_2	1	0	6	1	1
a_3	5	8	15	9	10
	7	5	3	2	

14. (a) Find the assignment that minimize the total unit cost.

	M_1	M_2	M_3
J_1	19	28	31
J_2	11	17	16
J_3	12	15	13

Or

(b) Solve the assignment problem

	A	B	C	D
X	18	24	28	32
Y	8	13	17	19
Z	10	15	19	22

15. (a) Determine the optimum sequence for the 5 jobs and minimum total elapsed time. Find also the idle time of machines M_1 and M_2.

Job	A	B	C	D	E
Machine M_1	5	4	8	7	6
Machine M_2	3	9	2	4	10

Or

(b) Determine the optimum sequence for the 8 jobs and minimum total elapsed time. Find also the idle time of machines M_1 and M_2.

Job	1	2	3	4	5	6	7	8
Machine M_1	14	26	17	11	9	26	18	18
Machine M_2	21	15	16	21	22	12	13	25

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve the following LPP by simplex method.

Minimize $z = x_1 - 3x_2 + 2x_3$

Subject to

$3x_1 - x_2 + 2x_3 \leq 7$

$-2x_1 + 4x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

$x_1, x_2, x_3 \geq 0.$

Or

(6 pages) **Reg. No. : ....**

Code No. : 40336 E **Sub. Code : JMMA 41/ JMMC 41**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Fourth Semester

Mathematics/Mathematics with CA – Main

ABSTRACT ALGEBRA

(For those who joined in July 2016 only)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. In $R*$, if $a*b=\frac{ab}{2}$, then the identity element is ———— .

   (a) 1 (b) 0

   (c) 2 (d) $\frac{1}{2}$

2. In the group $(C*, \cdot)$, order of i is ———— .

   (a) 1 (b) 2

   (c) 3 (d) 4

3. If $\alpha = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$, then $\alpha\beta =$

(a) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$

4. If P is a prime number and $(a, p) = 1$, then $a^{p-1} \equiv$ ———— (mod p).

(a) 0 (b) 1

(c) a (d) p

5. Any infinite cyclic group is isomorphic to ————.

(a) $(Z_n, \oplus)$ (b) $(R, +)$

(c) $(Q, +)$ (d) $(Z, +)$

6. Aut $Z_8 \cong$ ————.

(a) V_4 (b) $(Z_2, \oplus)$

(c) $(Z, +)$ (d) $(Z_8, \oplus)$

7. The characteristic of an integral domain is

(a) 1 (b) Prime number

(c) (a) or (b) (d) 0 or 1

8. R is a Boolean ring if ———— for all $a \in R$.

(a) $a^2 = a$

(b) $a^2 = e$

(c) $a + a = 0$

(d) $a^2 + a = 0$

9. The units in $Z[x]$ are

(a) 0, 1 (b) 1, −1

(c) 0, −1 (d) 0, 1 −1

10. A field of quotients of Z is

(a) C (b) R

(c) Q (d) Z

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Answer should not exceed 250 words.

11. (a) Let G be a group and $a \in G$. Suppose order of a is n. Then show that $a^m = e$ if and only if n divides m.

Or

(b) Show that the centre of a group G is a subgroup of G.

12. (a) State and prove Lagrange's theorem.

Or

(b) State and prove Euler's theorem.

13. (a) Show that if a group G has exactly one subgroup H of given order then H is a normal subgroup of G.

Or

(b) Show that a homomorphism $f: G \to G'$ is one–one if and only if $Ken\ f = \{e\}$.

14. (a) If $f: Q \to Q$ is an isomorphism, then prove that f is the identify map.

Or

(b) Prove the following :

(i) Z_n is an integral domain $\Leftrightarrow n$ is a prime number.

(ii) The characteristics of an integral domain is either 0 or a prime number.

15. (a) If R is a ring, show that $R[x]$ is also a ring.

Or

(b) Check that whether the function $f: C \to m_2(R)$ defined by $f(a+ib) = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ is a homomorphism.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

Answer should not exceed 600 words.

16. (a) Show that the union of two subgroups of a group is a subgroup if and only if one is contained in the other.

Or

(b) Let A and B be two subgroups of a group G. Then prove : AB is a subgroup of $G \Leftrightarrow AB = BA$.

17. (a) Show that a subgroup of a cyclic group is cyclic.

Or

(b) Let H and K be two finite subgroups of a group G. then show that $|HK| = \frac{|H||K|}{|H \cap K|}$.

18. (a) State and prove the fundamental theorem of Homorphism for groups.

Or

(b) Prove that $I(G)$ is a normal subgroup of Aut G.

19. (a) If R is a ring such that $a^2 = a$ for all $a \in R$, prove that.

(i) $a + a = 0$

(ii) $a + b = 0 \Rightarrow a = b$

(iii) $ab = ba$

Or

(b) Show that every P.I.D is a U.F.D.

20. (a) State and prove the Division algorithm.

Or

(b) Show that any integral domain can be embedded in a field.

———————

(6 pages) **Reg. No. :** ....

Code No. : 40570 E **Sub. Code : SMMA 41**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Fourth Semester

Mathematics — Main

ABSTRACT ALGEBRA —I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. In $\mathbb{R}^*$, define $a * b = \frac{ab}{2}$. Then the identity is

(a) 1 (b) 0

(c) 2 (d) 1/2.

2. Order of (-2) in the group $(\mathbb{Z}, +)$ is

(a) 0 (b) 2

(c) 1 (d) ∞.

3. Let G be a finite group and H be a subgroup of G. If $[G : H] = 1$ then $H =$

(a) $\{e\}$ (b) G

(c) e (d) $\{1\}$.

4. In the group $(Z_7 - \{0\}, \odot)$, $\langle 2 \rangle =$

(a) $Z_7 - \{0\}$ (b) Z_7

(c) $\{1, 2, 4\}$ (d) $\{2, 4, 6\}$.

5. Order of the quotient group $Z_6 / \langle 3 \rangle$ is

(a) 2 (b) 3

(c) 6 (d) 1.

6. The kernel of the homomorphism $f : (\mathbb{Z}, +) \to (\mathbb{R}^*, \cdot)$ given by $f(x) = 3^x$ is

(a) $\{1\}$ (b) $\{3\}$

(c) $\{0\}$ (d) $\{-1, 1\}$.

7. The characteristic of the ring $(Q, +, \cdot)$ is

(a) 0 (b) ∞

(c) 4 (d) 6.

8. Z_{12} is not an integral domain because

(a) 12 is not a prime

(b) 4 is a zero-divisor

(c) (a) and (b)

(d) there are zero-divisors.

9. Let $f(x), \quad g(x) \in z_4[x]$ be defined as $f(x) = x^2 + 2x + 3$ and $g(x) = 3x^2 + 2x + 2$. Then degree of $[f(x) + g(x)]$ is

(a) 0 (b) 2

(c) 4 (d) 1.

10. Field of quotients of Q is

(a) N (b) $\mathbb{Z}$

(c) $\mathbb{R}$ (d) $\mathbb{C}$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let G be a group and $a \varepsilon G$. Let $H_a = \{x / x \in G \text{ and } ax = xa\}$. Show that H_a is a subgroup of G.

Or

(b) Let n be a prime. Then prove that $(Z_n - \{0\}, \odot)$ is a group.

12. (a) State and prove Fermat's theorem.

Or

(b) Let H be a subgroup of a group G. Then prove the following

(i) $a \varepsilon H \Leftrightarrow aH = H$.

(ii) $aH = bH \Leftrightarrow a^{-1} b \varepsilon H$.

13. (a) Prove that a subgroup N of G is normal iff the product of two right cosets of N is again a right coset of N.

Or

(b) Show that any finite cyclic group of order 'n' is isomorphic to $(Z_n, \oplus)$.

14. (a) Show that a ring R has no zero-divisors iff cancellation law is valid in R.

Or

(b) Let R be a commutative ring with identity. Prove that R is a field iff R has no proper ideals.

15. (a) Show that the Kernel of a homomorphism is an ideal.

Or

(b) Prove that $R[x]$ is an integral domain iff R is an integral domain.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that the union of two subgroups of a group G is a subgroup iff one is contained in the other.

Or

(b) (i) Let G be a group and 'a' be an element of order n in G. Then prove that $a^m = e$ iff n divides m.

(ii) Let G be a group and 'a' be an element of order 'n' in G. Then prove that the order of a^s, where $0 < s < n$, is $\frac{n}{d}$ where d is the g.c.d. of n and s.

17. (a) Prove that a group G has no proper subgroups iff it is a cyclic group of prime order.

Or

(b) State and prove Lagrange's theorem.

18. (a) For any group G, prove the following :

(i) $(Aut\, G, \circ)$ is a group.

(ii) $I(G)$ is a normal subgroup of $Aut\, G$.

Or

(b) Let A_n be the set of all even permutations in S_n. Prove that A_n is a group containing $\frac{n!}{1}$ permutations.

19. (a) (i) Prove that a finite commutative ring with out zero-divisors is a field.

(ii) Prove that any field is an integral domain.

Or

(b) (i) Define an ideal, maximal ideal and prime ideal.

(ii) Let R be a commutative ring with identity. Prove that an ideal P of R is prime $\Leftrightarrow R/P$ is an integral domain.

20. (a) State and prove division algorithm.

Or

(b) Prove that the only isomorphism $f : Q \to Q$ is the identity map.

———————

(6 pages) **Reg. No. : ....**

Code No. : 40353 E **Sub. Code : JSMA 4 A/ JSMC 4 A**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Fourth Semester

Mathematics/Mathematics with CA — Main

Skill Based Subject — TRIGONOMETRY, LAPLACE TRANSFORMS AND FOURIER SERIES

(For those who joined in July 2016 only)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. $\cos 4\theta + 4\cos 2\theta + 3 =$ ————

   (a) $2^3 \cos^4 \theta$ (b) $\cos^4 \theta$

   (c) $2^4 \cos^4 \theta$ (d) $2\cos^4 \theta$

2. $16\sin^4 \theta - 20\sin^2 \theta + 5 =$ ————

   (a) $\sin 5\theta$ (b) $\frac{\sin 5\theta}{\sin \theta}$

   (c) $\sin 4\theta$ (d) $\frac{\sin 4\theta}{\sin \theta}$

3. $\cos(ix) =$ ______________

(a) $\cos x$ (b) $i\cos x$

(c) $\cos h\, x$ (d) $i\cos h\, x$

4. $Log(-1) =$ ______________

(a) $i2n\pi$ (b) $-i2n\pi$

(c) 0 (d) $i(2n+1)\pi$

5. $L(t\cos t) =$ ______________

(a) $\frac{1}{s^2+1}$ (b) $\frac{s^2}{s^2+1}$

(c) $\frac{s^2-1}{s^2+1}$ (d) $\frac{s^2-1}{(s^2+1)^2}$

6. $L^{-1}\left(\frac{1}{s^2}\right) =$ ______________

(a) 1 (b) $\frac{1}{t}$

(c) t (d) t^2

7. If $y(0) = y'(0) = 0$, then $L(y'') =$

(a) 0 (b) 1

(c) $s^2 L(y)$ (d) $s L(y)$

8. $L^{-1}\left(\frac{1}{s-2}\right)=$

(a) $t-2$ (b) e^{2t}

(c) e^{-2t} (d) $2e^{t}$

9. If $f(x)$ is an even function, then

(a) $f(x)=f(-x)$ (b) $f(x)=-f(-x)$

(c) $f(x)=f(x^2)$ (d) $f(x)=f(f(x))$

10. If $f(x)$ is an odd function defined in $(-l, l)$, then in the Fourier expansion $a_0 =$ ————

(a) 0 (b) l

(c) $2l$ (d) $\frac{\pi}{2}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Answer should not exceed 250 words.

11. (a) Expand $\cos 5\theta$ interms of powers of $\cos\theta$.

Or

(b) Prove: $2^5\cos^6\theta=\cos 6\theta+6\cos 4\theta+15\cos\theta+10$.

12. (a) Prove: $\cos h^{-1} x = \log_e\left(x+\sqrt{x^2-1}\right)$.

Or

(b) Seperate into real and imaginary parts: $\tan^{-1}(x+iy)$.

13. (a) Find: $L\left(\frac{1-e^t}{t}\right)$.

Or

(b) Find: $L^{-1}\left(\frac{1}{(s+1)(s^2+2s+2)}\right)$.

14. (a) Solve: $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=4e^{-t}$ given that $y=0, \frac{dy}{dt}=0$ when $t=0$.

Or

(b) Solve: $\frac{d^2y}{dt^2}+4y=A\sin kt$ given that $y=\frac{dy}{dt}=0$ when $t=0$.

15. (a) Find the Fourier sine series of $f(x)=x$ in $0<x<2$.

Or

(b) Find the Fourier expansion of the function

$$f(x)=\begin{cases}\pi+2x, & -\pi<x<0\\ \pi-2x, & 0\le x\le\pi\end{cases}$$

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

Answer should not exceed 600 words.

16. (a) Prove : $\frac{\sin 7\theta}{\sin\theta} = 7 - 56\sin^2\theta + 112\sin^4\theta - 64\sin^6\theta$.

Or

(b) Expand $\cos^5\theta\sin^3\theta$ in a series of sires of multiples of θ.

17. (a) If $\sin(\theta + i\phi) = \tan\alpha + i\sec\alpha$, then show that $\cos 2\theta \cos h\, 2\phi = 3$.

Or

(b) Prove : $\frac{\sin\theta}{1!} + \frac{\sin 2\theta}{2!} + \ldots = e^{\cos\theta}\sin(\sin\theta)$.

18. (a) Prove:

(i) $L(tf(t)) = \frac{-d}{ds}F(s)$

(ii) $L\left(\frac{f(t)}{t}\right) = \int_s^\infty f(s)\,ds$.

Or

(b) Find

(i) $L^{-1}\left(\frac{s+2}{\left(s^2+4s+5\right)^2}\right)$

(ii) $L^{-1}\left(\frac{s}{\left(s+3\right)^2+4}\right)$.

19. (a) Solve: $\frac{d^2y}{dt^2}+2\frac{dy}{dt}-3y=\sin t$; given that $y=\frac{dy}{dt}=0$ when $t=0$.

Or

(b) Solve : $\frac{dx}{dt}-\frac{dy}{dt}-2x+2y=1-2t$;

$\frac{d^2x}{dt^2}+2\frac{dy}{dt}+x=0$; given that

$x=0,\ y=0,\ \frac{dx}{dt}=0$ when $t=0$.

20. (a) Find the Fourier series of $f(x)=x^2$ in $(-\pi,\pi)$.

Or

(b) Find the Fourier cosine series of $f(x)=\pi-x$ in $(0,\pi)$.

Reg. No. : ....

Code No. : 40583 E **Sub. Code : SNMA 4 B**

U.G. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Fourth Semester

Mathematics

Non–Major – Elective – FUNDAMENTALS OF STATISTICS — II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. For given 'n' attributes, the total number of class frequencies is

(a) 3^n

(b) 2^n

(c) 3^2

(d) $3^n - 1$

2. A survey reveals that out of 1000 people in a locality, 800 like coffee; 700 like tea; 660 like both coffee and tea. The number of people liking neither coffee nor tea is

(a) 40 (b) 100

(c) 160 (d) 200

3. If the Laspeyre's and Paasche's index number are respectively m and n, then the Fisher's index number is

(a) $\frac{m+n}{2}$ (b) $\sqrt{mn}$

(c) $\frac{2mn}{m+n}$ (d) mn

4. With usual notations, Laspeyre's index number is

(a) $\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$ (b) $\frac{\Sigma p_0 q_0}{\Sigma p_1 q_0} \times 100$

(c) $\frac{\Sigma p_0 q_1}{\Sigma p_1 q_1} \times 100$ (d) $\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$

5. The arithmetic mean of Laspeyre's and Paasche' index number is defined to be

(a) Marshall-Edgeworth's index number

(b) Fisher's index number

(c) Fixed base index number

(d) Bowley's index number

6. With the usual notations, Marshall's index number is

(a) $$\frac{\Sigma p_1 q_0 + \Sigma p_1 q_1}{\Sigma p_0 q_0 + \Sigma p_0 q_1} \times 100$$

(b) $$\frac{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} + \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100}{2}$$

(c) $$\sqrt{\frac{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} + \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}{2}} \times 100$$

(d) $$\sqrt{\frac{\Sigma p_1 q_0 + \Sigma p_1 q_1}{\Sigma p_0 q_0 + \Sigma p_0 q_1}} \times 100$$

7. With the usual notations, $\Sigma p_1 q_0 = 433.5$, $\Sigma p_1 q_1 = 466.5$, $\Sigma p_0 q_0 = 343$ and $\Sigma p_0 q_1 = 370$, Fisher's index number is

(a) 126.2 (b) 128

(c) 125 (d) 125.2

8. ———— index number satisfies time reversal test.

(a) Bowley (b) Marshall

(c) Fisher's (d) Fixed base

9. For fitting a straight line, $y = ax + b$ the normal equations are

(a) $a\Sigma x_i^2 + b\Sigma x_i = \Sigma x_i y_i$ and $a\Sigma x_i + nb = \Sigma y_i$

(b) $b\Sigma x_i^2 + a\Sigma x_i = \Sigma x_i y_i$ and $b\Sigma x_i + na = \Sigma x_i$

(c) $a\Sigma x_i^2 + b\Sigma x_i = \Sigma x_i y_i$ and $b\Sigma x_i + na = \Sigma x_i$

(d) $b\Sigma x_i^2 + a\Sigma x_i = \Sigma x_i y_i$ and $a\Sigma x_i + nb = \Sigma y_i$

10. The principle of ———— states that the parameters involved in $f(x)$ should be chosen in such a way that $\sum_{i=1}^{n} d_i^2$ is minimum where $d_i = y_i - f(x_i)$

(a) most squares (b) least squares

(c) atleast square (d) none of these

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Given the following ultimate class frequencies. Find the frequencies of the positive and negative classes and the total number of observations.

$(AB) = 733$, $(A\beta) = 840$; $(\alpha B) = 699$, $(\alpha\beta) = 783$.

Or

(b) Given $(A)=30$, $(B)=25$, $(\alpha)=30, (\alpha\beta)=20$. Find :

(i) N.

(ii) (β).

(iii) (AB).

12. (a) Find Laspeyre's index number for the following data :

Commodities	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	2	8	4	6
B	5	10	6	5
C	4	14	5	10
D	2	19	2	13

Or

(b) Find Paasche"s index number :

Commodities	Base Year		Current Year	
	Price	Quantity	Price	Quantity
Wheat	10	2	20	5
Rice	30	4	50	8

13. (a) Find Bowley's index number.

Items	Base Year		Current Year	
	Price	Quantity	Price	Quantity
TV	15 0	3	500	1
Computer	200	2	200	1

Or

(b) Find Marshall Edgeworth's index number.

Commodities	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	50	2	100	3
B	30	3	120	7
C	10	1	50	2

14. (a) Prove that the Fisher's index number I_{01} is an ideal index number $I_0 \times I_{10} = 1$.

Or

(b) Find Fisher's index number for the year 1992 data given below :

Year	Rice		Wheat		Flour	
	Price	Quantity	Price	Quantity	Price	Quantity
1988	9.3	100	6.4	11	5.1	5
1992	4.5	90	3.7	10	2.7	3

15. (a) Fit a straight line to the following data :

x :	0	1	2	3	4
y :	1	1.8	3.3	4.5	6.3

Or

(b) Fit a straight line to the following data :

x :	0	1	2	3	4
y :	2.1	3.5	5.4	7.3	8.2

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) In a class test in which 135 candidates were examined for proficiency in English and Maths. It was discovered that 75 students failed in English, 90 failed in Maths and 50 failed in both. Find how many candidates

(i) have passed in Maths.

(ii) have passed in English, failed in Maths.

(iii) have passed in both.

Or

(b) If $(A)=(\alpha)=(B)=(\beta)=N/2$, show that

(i) $(AB)=(\alpha\beta)$ and

(ii) $(A\beta)=(\alpha B)$.

17. (a) Find the missing price in the following data if the ratio between Laspeyre's and Pasche's index number is 25 : 24.

Commodities	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	1	15	2	15
B	2	15	—	30

Or

(b) Calculate Laspeyre's and Paasche's index numbers for the following data :

Commodities	Base Year 1990		Current Year 1992	
	Price	Quantity	Price	Quantity
A	2	10	3	12
B	5	16	6.5	11
C	3.5	18	4	16
D	7	21	9	25
E	3	11	3.5	20

18. (a) For the data given below, find Bowley's index number.

Commodities	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	6	50	10	56
B	2	100	2	120
C	4	60	6	60
D	10	30	12	24
E	8	40	12	26

Or

(b) Find Marshall's index number for the following data :

Commodities	2015		2018	
	Price	Quantity	Price	Quantity
A	20	8	40	6
B	50	10	60	5
C	40	15	50	15
D	20	20	20	25

19. (a) Show that the following data satisfies time reversal test.

Items	2015		2017	
	Price	Quantity	Price	Quantity
A	6	50	10	56
B	2	100	2	120
C	4	60	6	60
D	10	30	12	24
E	8	40	12	36

Or

(b) Compute Fisher's index number for the following :

Year	Tomato		Brinjal		Onion	
	Price	Quantity	Price	Quantity	Price	Quantity
1980	4	50	3	10	2	5
1990	10	40	8	8	4	4

20. (a) Show that the line of best fit to the following data is $y - 8 - 0.5x$.

x :	6	7	7	8	8	8	9	9	10
y :	5	5	4	5	4	3	4	3	3

Or

(b) Fit a straight line to the following data :

x :	1	2	3	4	6	8
y :	2.4	3	3.6	4	5	6

(6 pages) **Reg. No. : ....**

Code No. : 40342 E Sub. Code : JMMA 63/ JMMC 63

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Sixth Semester

Mathematics/Mathematics with CA – Main

NUMBER THEORY

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. If n is an odd integer and $r = \frac{1}{2}(n-1)$, then

(a) $\binom{n}{r} = \binom{n}{r+1}$ (b) $\binom{n}{r} = \binom{n+1}{r+1}$

(c) $\binom{n}{r} = \binom{n}{r-1}$ (d) $\binom{n+1}{r} = \binom{n+1}{r+1}$

2. $\binom{6}{3} + \binom{6}{4} =$ __________

(a) $\binom{7}{4}$ (b) $\binom{6}{7}$

(c) $\binom{2}{7}$ (d) $\binom{7}{6}$

3. gcd (8, 36) = __________.

(a) -4 (b) 4

(c) -2 (d) 2

4. If $lcm\,(a, b) = ab$, then gcd $(a, b) =$ __________

(a) a (b) b

(c) ab (d) 1

5. For $n \geq 2$, $\sqrt[n]{n}$ is

(a) irrational (b) rational

(c) composite (d) integer

6. How many prime numbers are of the form $n^2 - 4$?

(a) 1 (b) 2

(c) 3 (d) 4

7. The remainder when 2^{50} is divided by 7 is

(a) 1 (b) 2

(c) 3 (d) 4

8. Number of solutions of $9x \equiv 21 \pmod{30}$ is

(a) 1 (b) 2

(c) 3 (d) 4

9. If $a^n = a \pmod{n}$, then n is called a ————.

(a) prime number

(b) pseudo prime number

(c) composite number

(d) twin prime number

10. The unit digit of 3^{100} is

(a) 0 (b) 1

(c) 2 (d) 3

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) State and prove first principle of Induction.

Or

(b) Prove that $1^2 + 2^2 + ... + n^2 = \frac{n(2n+1)(n+1)}{6}$.

12. (a) If $\gcd(a, b) = 1$, then prove that $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.

Or

(b) State and prove Euclid's lemma.

13. (a) State and prove Euclid's theorem.

Or

(b) Prove that $\sqrt{2}$ is an irrational number.

14. (a) Find the remainder when $1! + 2! + \ldots + 99! + 100!$ is divided by 12.

Or

(b) Show that 41 divides $2^{20} - 1$.

15. (a) State and prove Fermat's theorem.

Or

(b) State and prove Wilson's theorem.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Establish the binomial theorem by induction.

Or

(b) State and prove Archimedean property.

17. (a) State and prove Euclidean algorithm.

Or

(b) State and prove the division algorithm.

18. (a) State and prove Fundamental theorem of Arithmetic.

Or

(b) Explain the sieve of Eratosthenes.

19. (a) Solve : $17x \equiv 9 \pmod{276}$.

Or

(b) State and prove the Chinese Remainder *theorem*.

20. (a) If p is a prime, then $a^p \equiv a \pmod{p}$ for any integer a.

Or

(b) Prove that the quadratic congruence $x^2 + 1 \equiv 0 \pmod{p}$, where p is an odd prime has a solution iff $p \equiv 1 \pmod{4}$.

(7 pages)

Reg. No. : ....

Code No. : 7112 **Sub. Code : PMAM 11**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

First Semester

Mathematics — Core

ALGEBRA — I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If ϕ is a homomorphism then $\phi(ab) =$ ————.

   (a) $\phi(a)\,\phi(b)$ (b) $\phi(a/b)$

   (c) $\phi(a) - \phi(b)$ (d) none

2. Let G be a graph of order 99 and let H be a subgroup of order 11. Then H contains a normal subgroup $N \neq (e)$ or order

   (a) 3 or 5 (b) 3 or 6

   (c) 11 (d) 9

3. If G is a group and H is a subgroup of index 2 in G then

(a) H is a normal subgroup of G

(b) H is abelian subgroup of G

(c) $aHa^{-1} \neq H$

(d) none of these

4. If G is abelian of order $O(G)$ and $p^{\alpha} \mid O(G)$, $p^{\alpha+1} \nmid O(G)$, then

(a) there is a subgroup of G of order p^{α}

(b) there is a unique subgroup of G of order p^{α}

(c) there must be a subgroup of G of order $> p^{2}$

(d) none of these

5. Product of two odd permutations is an ———— permutation.

(a) odd (b) even

(c) both (d) none

6. Let $a \in Z$, centre of G. Then

(a) $N(a) = G$ (b) $N(a) \neq G$

(c) $O(Z) = p^{n}$ (d) none of these

7. The number of p-sylow subgroups in G, for a given prime is of the form

(a) $1 + kp$ (b) p^a

(c) $kp + 3$ (d) none

8. S_{p^k} has a p-sylow subgroup of order

(a) $p^{n(k)}$ (b) p^k

(c) p (d) none

9. Every finite abelian group is the direct product of ________.

(a) subgroups (b) abelian groups

(c) cyclic groups (d) none

10. If A and B are groups then $A \times B$ is isomorphic to

(a) A (b) B

(c) $B \times A$ (d) None

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that the subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

Or

(b) If ϕ is a homomorphism of G into $\overline{G}$ with Kernel K, then prove that K is a normal subgroup of G.

12. (a) If G is a group and N is a normal subgroup of G such that both N and G/N are solvable, prove that G is solvable.

Or

(b) Prove that any non abelion group of order 6 is isomorphic to S_3.

13. (a) If $o(G) = p^2$, where p is a prime number, then show that G is abelian.

Or

(b) If G is a finite group, then show that $C_a = \frac{o(G)}{o(N(a))}$, where C_a is the number of elements conjugate to a in G.

14. (a) Show that the number of p-sylow subgroups in G equal index of $N(P)$ in G.

Or

(b) If A, B are finite subgroups of a group G, then prove that $O(AxB) = \frac{O(A)\,O(B)}{O\left(A \cap xBx^{-1}\right)}$.

[P.T.O.]

15. (a) Let G be a group and suppose that G is the internal direct product of $N_1, N_2, \ldots, N_n$, let $T = N_1 \times N_2 \times \ldots \times N_n$. Then prove that G and T are isomorphic.

Or

(b) Let G be a finite abelian group. Prove that G is isomorphic to the direct product of its sylow subgroups.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Sylow's theorem for abelian groups.

Or

(b) (i) Let H and K are finite subgroups of G orders $O(H)$ and $O(K)$ respectively then show that $O(HK) = \frac{O(H)\,O(K)}{O(H \cap K)}$.

(ii) If H and K are subgroups of G and $O(H) > \sqrt{O(G)}, O(K) > \sqrt{O(G)}$ then show that $H \cap K \neq (e)$.

17. (a) (i) Prove that a group is solvable if and only if $G^{(K)} = (e)$ for some integer K.

(ii) Prove that every homomorphic image of a solvable group is solvable.

Or

(b) If G is a group, H is a subgroup of G and S is the set of all right cosets of H in G. Then S show that there is a homomorphism θ of G into $A(S)$ and the Kernel of θ is the largest normal subgroup of G which is contained in H.

18. (a) Prove that the number of conjugate class in S_n is $p(n)$, the number of partions of n. Also prove that $a \in Z$ if and only if $N(a) = G$.

Or

(b) Define conjugate and also prove that conjugacy is an equivalence relation on G.

19. (a) State and prove Sylow's theorem for general group.

Or

(b) Find the number of 11 sylow subgroups and 13 sylow subgroups of a group or order $11^2 \times 13^2$ and show that this group is abelian.

20. (a) Prove that two abelian groups or oder p^n are isomorphic if and only if they have the same invariants.

Or

(b) Prove that every finite abelian group is a direct product of its cyclic groups.

(7 pages) **Reg. No. :** ....

Code No. : 7117 **Sub. Code : PMAM 21**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Second Semester

Mathematics – Core

ALGEBRA – II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 1 = 10 marks)

Answer ALL the questions.

Choose the correct answer :

1. If F is a field, the number of ideals of F is

   (a) 1 (b) 2

   (c) at least 2 (d) 0

2. The homomorphism ϕ of R into R' is an isomorphsim if and only if

   (a) $I(\phi) \neq (0)$ (b) $I(\phi)$ is an ideal of R

   (c) $I(\phi) = (0)$ (d) $I(\phi) = R$

3. If π is a prime element in the Euclidean ring R and $a \in R$ then

   (a) $(\pi, a) = 1$

   (b) a / π

   (c) If $\pi \nmid a$ then $(\pi, a) = 1$

   (d) If π / a then $(\pi, a) = 1$

4. A solution of $x^2 \equiv -1 (\text{mod} 13)$ is

   (a) 6 (b) 5

   (c) 3 (d) 2

5. The degree of $5 + 7x^2 + 4x^4 + 11x^5$ over the integers mod 11 is

   (a) 4 (b) 5

   (c) 0 (d) 2

6. Which one of the following is not true?

   (a) A Euclidean ring is a unique factorization domain

   (b) A Euclidean ring is a principal ideal ring

   (c) If F is a field, $F[x_1, x_2]$ is a principal ideal ring

   (d) If R is an integral domain then so is $R[x]$

7. If $R = \mathbb{Z}$, the ring of integers then rad R is

(a) $\mathbb{Z}$

(b) (0)

(c) (P) for same prime number P

(d) $2\mathbb{Z}$

8. The relation between rad R and Rad R is

(a) $Rad\ R = rad\ R$

(b) $Rad\ R \subseteq rad\ R$

(c) $rad\ R \subseteq Rad\ R$

(d) They are not comparable

9. A ring R is isomorphic to a sub direct sum of integral domains if and only if

(a) R is semi simple

(b) R is without prime radical

(c) R is a ring without identify

(d) R is a commutative ring

10. For any commutative regular ring R, $J(R)$ is

(a) ϕ (b) $\{0\}$

(c) R (d) the centre of R

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If U, V are ideals of R, let $U + V = \{u + v / u \in U, v \in V\}$. Prove that $U + V$ is also an ideal.

Or

(b) Let R be a commutative ring with unit element whose only ideals are (O) and R itself. Prove that R is a field.

12. (a) Let R be a Euclidean ring. Prove that any two elements a and b in R have a greatest common division and $d = \lambda a + \mu b$ for same $\lambda, \mu \in R$.

Or

(b) Prove that $J[i]$ is a Euclidean ring.

13. (a) If $f(x), g(x)$ are two new zero elements of $f[x]$, prove that $\deg(f(x)\ g(x)) = \deg f(x) + \deg g(x)$.

Or

(b) State and prove Gàuss's lemma.

14. (a) Let R be a principal ideal domain. Prove that R is semi simple if and only if R is either a field on has an infinite number of maximal ideals.

Or

(b) For any ring R, prove that the quotient ring $R / Rad\, R$ is without prime radical.

15. (a) Prove that an element a of the ring R is quasi regular if and only if there exists some $b \in R$ such that $a + b - ab = 0$.

Or

(b) Let R be a ring containing no non zero nil ideals. Prove that R is isomorphic to a sub direct sum of integral domain.

SECTION C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If U is an ideal of the ring R, prove that R/U is a ring and is a homomorsphic image or R.

Or

(b) Prove that every integral domain can be imbedded in a field.

17. (a) Prove that the ideal $A = (a_0)$ is a maximal ideal of the Euclidean ring R if and only if a_0 is a prime element or R.

Or

(b) After proving the necessary lemmas, prove that if p is a prime number of the form $4n+1$, then $p = a^2 + b^2$ for some integer a, b.

18. (a) State and prove the Eisenstein criterion.

Or

(b) If R is a unique factorization domain, prove that $R[x]$ is also a unique factorization domain.

19. (a) If I is an ideal of the ring R, prove that

(i) $rad(R/I) \supseteq \dfrac{rad\ R + I}{I}$ and

(ii) whenever $I \subseteq rad\ R$, $rad\left(\dfrac{R}{I}\right) =$ $(rad\ R)/I$.

Or

(b) Define a primary ring. Prove that a ring R is a primary ring if and only if R has a minimal prime ideal which contains all zero divisions.

20. (a) Prove that a ring R is isomorphic to a sub direct sum of ring Ri, if and only if R contains a collection of ideals $\{Ii\}$ such that $R/I_i \simeq R_i$ and $\bigcap I_i = (0)$.

Or

(b) Prove that every ring R is isomorphic to a sub direct of sum of sub directly irreducible rings.

(6 pages) **Reg. No. :** ....

Code No. : 7831 **Sub. Code : PMAM 11**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

First Semester

Mathematics — Core

ALGEBRA — I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. In the quotient group $\frac{G}{N}$, N is ————.
   (a) any proper subgroup of G
   (b) a cyclic subgroup of G
   (c) a normal subgroup of G
   (d) a proper abelian subgroup of G

2. The Kernal of a homomorphism $f : G \to G'$ ————.
   (a) a normal subgroup of G
   (b) $\{e\}$
   (c) a subgroup of G'
   (d) a normal subgroup of G'

3. The smallest non-abelian group is ————
   (a) S_3 (b) S_2
   (c) $\mathbb{Z}$ (d) $\mathbb{N}$

4. If G is a group of order 99 and H is a subgrou G of order 11 then $i(H)$ = ————.
   (a) 9 (b) 9!
   (c) 11 (d) 11!

5. The symmetric group S_n of order n is ————
   (a) a non-abelian group for any n
   (b) an abelian group for all n
   (c) non-abelian group only when $n \geq 3$
   (d) abelian group for $n = 3$

6. The group S_n has ———— elements.

(a) n (b) $n!$

(c) $n!/2$ (d) nC_2

7. Let G be a group of order 72. The number of 3-sylow subgroup is/are ————.

(a) 1 (b) 4

(c) either 1 or 4 (d) 0

8. Let $G = S_3$, ———— 2-Sylow subgroup of order 2 in G.

(a) 1 (b) 2

(c) 4 (d) 3

9. The number of non-isomorphic abelian groups of order 2^4 is ————.

(a) 4 (b) 5

(c) 7 (d) 1

10. Every finite abelian group is the direct product of ———— groups.

(a) normal (b) cyclic

(c) sub (d) none of these

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If ϕ is a homomorphism of G into $\overline{G}$ with Kernal K, then prove that K is a normal subgroup of G.

Or

(b) Let ϕ be a homomorphism of G onto $\overline{G}$ with Kernal K, then prove that $G/K \cong \overline{G}$.

12. (a) Prove that $I(G) \approx G/Z$, where $I(G)$ is the group of inner automorphisms of G and Z is the center of G.

Or

(b) Prove that a subgroup of a solvable group is solvable.

13. (a) Prove that conjugacy is an equivalence relation on G.

Or

(b) State and prove Cauchy's theorem.

14. (a) Prove that $n(k) = 1 + p + p^2 + \ldots + p^{k-1}$ $n(k)$ defined by $p^{n(k)} / p^{(k)}!$ but $p^{n(k)+1} \times p^{(k)}!$

Or

(b) If A, B are finite subgroups of G then prove that $O(A\,x\,B) = \frac{O(A)\,O(B)}{O\left(A \cap x\,Bx^{-1}\right)}$.

15. (a) Suppose that G is the internal direct product of $N_1, N_2, \ldots, N_n$ then prove that for $i \neq j$, $N_i \cap N_j = (e)$ and if $a \in N_i, b \in N_j$ then $ab = ba$.

Or

(b) Let G be a finite abelian group. Prove that G is isomorphic to the direct product of its sylow subgroups.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If H and K are finite subgroups of G of orders $O(H)$ and $O(K)$. Prove that $O(HK) = \frac{O(H)\,O(K)}{O(H \cap K)}$.

Or

(b) Prove that HK is a subgroup of G if and only if $HK = KH$.

17. (a) State prove Cayley's theorem.

Or

(b) Prove that A group G is solvable if and only if $G^{(K)} = (e)$ for some $K \geq 1$.

18. (a) If G is a finite group prove that $C_a = O(G)\,|\,O\,CN(a)$.

Or

(b) If $O(G) = p^n$ where p is a prime number then prove that $Z(G) \neq (e)$.

19. (a) If p is a prime number and $p^\alpha / O(G)$ then prove that G has a subgroup of order p^α.

Or

(b) State and prove second part of Sylow's theorem.

20. (a) Prove that every finite abelian group is the direct product of cyclic groups.

Or

(b) Show that every group of order p^2, p a prime, is either cyclic or is isomorphic to the direct product of two cyclic groups each of order p.

———————

(6 pages) **Reg. No. :** ....

Code No. : 7834 **Sub. Code : PMAM14**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. Which of the following is the Non-Homogeneous Equation?

   (a) $y''=0$ (b) $y'=y$

   (c) $y''=y$ (d) $y'=e^x$

2. Any linear combination of two solutions of the homogeneous equation $y''+P(x)y'+Q(x)y=0$ is also a ————

   (a) solution (b) equation

   (c) IVP (d) BVP

3. Which of the following is the Transcendental Equation?

   (a) $x=0$ (b) $y=0$

   (c) $z=0$ (d) $e^x=0$

4. Any point that is not ordinary point of the equation $y''+P(x)y'+Q(x)y=0$ is called

   (a) Singular point

   (b) Special function piont

   (c) Ordinary point

   (d) Point function

5. $P_n=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$ is called ———— formula.

   (a) Legendre (b) Rodrigues

   (c) Binomial (d) Bessel

6. $y=a_0x^m+a_1x^{m+1}+....$ is called ———— series.

   (a) Frobenius (b) Rodrigues

   (c) Binomial (d) Bessel

7. $\Gamma(6)=$ ————

   (a) 20 (b) 120

   (c) 100 (d) 40

8. $\Gamma\left(\frac{5}{2}\right) =$ ——————

(a) 1.32 (b) 2.32

(c) 0 (d) ∞

9. If W(t) is the Wronskian of the two solutions of the homogeneous system then W(t) is —————— on [a,b].

(a) Identically Zero (b) Never zero

(c) either (a) or (b) (d) zero

10. The system $\frac{dx}{dt} = a(t)x + f(t), f(t) = 0$ then this system is called ——————

(a) Homogeneous (b) non-Homogeneous

(c) non-linear (d) Wronskian

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Solve $y''+y'=0$.

Or

(b) Find the particular solution for $y''+y = \csc x$.

12. (a) Prove that the equation $(t-2)x''+x=0$ does not has an ordinary point t = 2.

Or

(b) Find the general solution for $y''+y=0$.

13. (a) Determine the nature of Singularity of $f(z) = \frac{e^z}{z}$.

Or

(b) Discuss the nature of Singularity of $f(z) = \frac{1}{\sin(\cos z)}$.

14. (a) Prove that $\frac{d}{dt}\left[t^{-p}T_{p+1}(t)\right] = -\left[t^{-p}T_{p+1}(t)\right]$

Or

(b) Find the general solution of the equation

$$9x^2y''+9xy'+\left(9x^2-\frac{1}{4}\right)y=0.$$

15. (a) Find the Wronkian value W of the equation

$$\frac{d^2y}{dx^2}+4y=\tan 2x$$

Or

(b) Find the Complementary function for the differential equation.

$$x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-3y=x^2\log x.$$

SECTION C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If $y_1(x)$ and $y_2(x)$ are two solutions of $y''+P(x)y'+Q(x)y=0$ on [a,b], then prove that their Wronskian $W=W(y_1,y_2)$ is either identically zero or never zero on [a,b].

Or

(b) Show that $y=C_1\sin x+C_2\cos x$ is the general solution of $y''+y'=0$ on any interval, and find the particular solution for which $y(0)=Z \; and \; y'(0)=3$.

17. (a) Find the power series solution for the equation $y'=t^2-y^2, y(0)=0$ for $t=0$.

Or

(b) Find the power series solution for the equation $(1+x)y'=Py, y(0)=1$.

18. (a) Consider the equation $t(t-1)^2(t+3)x''+t^2x'-(t^2+t-1)x=0$. Check whether the point t = 1 is the regular Singular point or not.

Or

(b) If P_n is the Legendre polynomial, the prove that $\int_{-1}^{1} P_n^2(t)dt=\frac{2}{2n+1}$.

19. (a) Consider the differential equation $4x^2y''+4xy'+\left(x-\frac{1}{36}\right)y=0$. Set $Z=\sqrt{x}$ and reduce the differential equation to a Bessel equation in $Z, \frac{dy}{dz} \; and \frac{d^2y}{dz^2}$.

Or

(b) Prove that $P_n'(1)=\frac{1}{2}n(n+1)$.

20. (a) Find the general solution of $\frac{dx}{dt}=x+y; \frac{dy}{dt}=4x-2y$.

Or

(b) Find the general solution of $\frac{dx}{dt}=3x-4y; \frac{dy}{dt}=x-y$.

(8 pages) **Reg. No. : ....**

Code No. : 7832 **Sub. Code : PMAM 12**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2019.

First Semester

Mathematics – Core

ANALYSIS – I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — $(10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. The set of all subsequential limits of a sequence in a metric space X from a ———— subset of X.

   (a) open

   (b) closed

   (c) countable

   (d) perfect

2. A metric space is called separable if it contains a ________ dense subset.

(a) countable (b) uncountable

(c) perfect (d) none of these

3. $\lim_{n \to \infty} \left[1 + \frac{1}{n}\right]^n =$ ________.

(a) 0 (b) 1

(c) e (d) None of these

4. The series $\sum \frac{(-1)^n}{n}$ is ________.

(a) converges

(b) diverges

(c) converges absolutely

(d) none of these

5. If the series $\sum |a_n|$ converges then the series $\sum a_n$ is said to ________.

(a) diverges

(b) converges

(c) converges absolutely

(d) converges non absolutely

6. Let $\alpha = \limsup_{n\to\infty} \sqrt[n]{a_n}$ then $\sum a_n$ diverges if ________.

(a) $\alpha = 1$ (b) $\alpha < 1$

(c) $\alpha > 1$ (d) $\alpha = 0$

7. Let f be monotonic on (a, b) then the set of points of (a, b) at which f is discontinuous is ________.

(a) countable (b) uncountable

(c) atmost countable (d) none of these

8. Let f be a continuous mapping of a metric space X into a metric space Y then f is uniformly continuous on X if X is ________.

(a) connected (b) closed

(c) compact (d) none of these

9. Let f be defined by $f(x) = \begin{cases} x^2 \sin\frac{1}{x} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$

(a) f is differentiable at all points x

(b) f is differentiable at $x = 0$ and f is differentiable at other points

(c) f is not differentiable at all points x

(d) none of these

10. Suppose f is differentiable in (a, b) if ________ then f is monotonically increasing.

(a) $f'(x) = 0$

(b) $f'(x) \geq 0$

(c) $f'(x) \leq 0$

(d) None of these

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If E is an infinite subset of compact set K. Prove that E has a limit point in K.

Or

(b) Let K be a positive integer. If $\{I_n\}$ is a sequence of K-cells such that $I_n \supset I_{n+1}$, $n = 1, 2, 3, \ldots$ then prove that $\prod_{n=1}^{\infty} I_n$ is not empty.

12. (a) Prove that $\sum \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

Or

(b) Prove that e is irrational.

13. (a) State and prove Ratio test.

Or

(b) If $\sum a_n = A$ and $\sum b_n = B$ then prove that $\sum (a_n + b_n) = A + B$ and $\sum C\,a_n = CA$ for any fixed C.

14. (a) Prove that composition of continuous function is continuous.

Or

(b) Suppose f is a continuous mapping of a compact metric space X into a metric space Y, then prove that $f(X)$ is compact.

15. (a) Let f be defined by $f(x) = \begin{cases} x \sin \frac{1}{x} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$ prove that f is differentiable at all points except $x = 0$.

Or

(b) Let f be defined on $[a, b]$. If f is differentiable at a point $x \in [a, b]$ then prove that f is continuous at x.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Heine Borel theorem.

Or

(b) Prove that Cantor set is a perfect set.

17. (a) Prove that :

(i) $\lim_{n\to\infty} \sqrt[n]{n} = 1$.

(ii) If $p > 0$ and α is a real then $\lim_{n\to\infty} \frac{n^{\infty}}{(1+p)^{n}} = 0$.

Or

(b) Prove that if $\overline{E}$ is the closure of a set E in a metric space X then $diam\,\overline{E} = diam\,E$.

18. (a) Suppose :

(i) $\sum_{n=0}^{\infty} a_n$ converges absolutely

(ii) $\sum_{n=0}^{\infty} a_n = A$

(iii) $\sum_{n=0}^{\infty} b_n = B$

(iv) $C_n = \sum_{k=0}^{n} a_k b_{n-k} n = 0, 1, 2 \ldots$

then prove that $\sum_{n=0}^{\infty} C_n = AB$.

Or

(b) Prove that for any sequence $\{C_n\}$ of positive numbers, $\lim_{n \to \infty} \sqrt[n]{C_n} \leq \lim_{n \to \infty} \sup \frac{C_{n+1}}{C_n}$.

19. (a) Let f be monotonically increasing on (a, b) then prove that $f(x+)$ and $f(x-)$ exists at every point of x of (a, b). More precisely, $\sup f(t) = f(x-) \leq f(x) \leq f(x+) = \inf f(t)$

$x < t < b$.

Or

(b) Let f be continuous mapping of a compact metric space X into a metric space Y then prove that f is uniformly continuous on X.

20. (a) State and prove Taylor's theorem.

Or

(b) State and prove Chain rule for differentiation.

(6 pages) **Reg. No. :** ....

Code No. : 7113 **Sub. Code : PMAM 12**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

First Semester

Mathematics — Core

ANALYSIS – I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. If X is a metric space and $E \subset X$ then $E = \overline{E}$ if and only if E is

 (a) open (b) closed

 (c) perfect (d) bounded

2. The interval $[0\,1]$ is

 (a) countable (b) uncountable

 (c) bounded (d) cannot be defined

3. $\lim_{n\to\infty} \sqrt[2]{n}$ is

(a) 0 (b) 1

(c) ∞ (d) none

4. If $s_n = n^2$, then the sequence $\{s_n\}$ is

(a) bounded (b) convergent

(c) unbounded (d) none

5. The series $\frac{\sum(-1)^n}{n}$

(a) converges absolutely

(b) converges non absolutely

(c) diverges

(d) none

6. The radius of convergence for the series $\sum \frac{z^n}{n^2}$ is

(a) 1 (b) 0

(c) 2 (d) none

7. The function $f(x) = \begin{cases} x, & x \text{ rational} \\ 0 & x \text{ irrational} \end{cases}$ is continuous at

(a) every point

(b) every point other than 0

(c) $x = 0$

(d) every rational x

8. If f is a continuous mapping of a metric space X into a metric space Y. Then for any set $E \subset X$

(a) $\overline{f(E)} \subset f(\overline{E})$ (b) $\bar{f}(E) = f(E)$

(c) $\bar{f}(E) \subset F(E)$ (d) none

9. $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$ then $f'(0)$ is

(a) 0 (b) 1

(c) -1 (d) does not exist

10. If $f'(x) > 0$ in (a, b), then f is

(a) strictly increasing in (a, b)

(b) constant

(c) monotonically increasing in (a, b)

(d) none

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define a neighbourhood of a point P. Prove that every neighbourhood is an open set.

Or

(b) Prove that compact subsets of metric spaces are closed.

12. (a) If $\overline{E}$ is the closure of a set E in a metric space X, then prove that diam E = diam $\overline{E}$.

Or

(b) Prove that the sub sequential limit of a sequence $\{P_n\}$ in a metric space X form a closed subset of X.

13. (a) State and prove Root test.

Or

(b) For any sequence $\{C_n\}$ of positive numbers prove that $\limsup_{n\to\infty} \sqrt[n]{C_n} \le \limsup_{n\to\infty} \frac{C_{n+1}}{C_n}$.

14. (a) Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

Or

(b) If f is a continuous mapping of a metric space X into a metric space Y prove that $f(\overline{E}) \subset \overline{f(E)}$ for every set $E \subset X$.

15. (a) Let f be defined on $[a,b]$. If f has a local maximum at a point $x \in (a,b)$ and if $f'(x)$ exist then prove that $f'(x) = 0$.

Or

(b) Let f be defined for all real x and suppose that $|f(x) - f(y)| \le (x-y)^2$ for all real x and y. Prove that f is constant.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that every k-cell is compact.

Or

(b) Let p be a non empty perfect set in R^k. Then prove that p is uncountable.

17. (a) (i) Prove that $\Sigma \frac{1}{np}$ converges if $p > 1$ and diverges if $p \leq 1$.

(ii) Prove that $\sum_{n=3}^{\infty} \frac{1}{n \log n \log \log n}$ diverges.

Or

(b) Prove that

(i) $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$.

(ii) e is irrational.

18. (a) (i) State and prove ratio test.

(ii) State and prove Leibnitz theorem.

Or

(b) State and prove Merten's theorem.

19. (a) Let f be a continuous mapping of a compact metric space X into a metric space Y. Then prove that f is uniformly continuous on X.

Or

(b) Let E be a non compact set in R'. Then prove that

(i) there exists a continuous function on E which is not bounded.

(ii) there exists a continuous and bounded function on E which has no maximum.

(iii) If in addition, E is bounded then prove that there exists a continuous function on E which is not uniformly continuous.

20. (a) State and prove Taylor's theorem.

Or

(b) State and prove L' hospital's rule.

(7 pages) **Reg. No. :** ....

Code No. : 7118 **Sub. Code : PMAM 22**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Second Semester

Mathematics — Core

ANALYSIS – II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If P^* is a refinement of P then

   (a) $P \supset P^*$ (b) $P^* \supset P$

   (c) $P^* = P_1 \cup P_2$ (d) none

2. Which of the following is not true?

   (a) If f is continuous then $f \in R(\alpha)$ on $[a,b]$

   (b) If f is monotonic then $f \in R(\alpha)$ on $[a,b]$

   (c) If $f \in R(\alpha)$ then $|f| \in R(\alpha)$ on $[a,b]$

   (d) If $f, g \in R(\alpha)$ then $fg \in R(\alpha)$ on $[a,b]$

3. A continuous mapping γ of an interval $[a,b]$ into R^K is called a closed curve if

   (a) γ is one to one (b) $\gamma(a) = \gamma(b)$

   (c) $\gamma(a) \neq \gamma(b)$ (d) none

4. $\lim\limits_{m\to\infty} \lim\limits_{n\to\infty} (\cos m! \pi x)^{2n}$ is

   (a) Riemann integrable

   (b) Not Riemann integrable

   (c) Continuous

   (d) None

5. Every member of an equicontinuous family is

   (a) uniformly continuous

   (b) not uniformly continuous

   (c) discontinuous

   (d) none

6. If $f_n(x) = \dfrac{x^2}{x^2 + (1-nx)^2}$ $0 \le x \le 1$, $n = 1,2,3\ldots$ then $\{f_n\}$ is

   (a) pointwise bounded

   (b) uniformly bounded

   (c) unbounded

   (d) none

7. The set of all polynomials is

(a) an algebra (b) not an algebra

(c) unbounded (d) none

8. $\lim_{n\to\infty}\left(1-\frac{x}{n}\right)^n$ is

(a) e^x (b) e

(c) e^{-x} (d) none

9. An orthogonal system $\{\phi_n\}$ is orthonormal if $\int_a^b |\phi_n(x)|^2\,dx$ is

(a) 1 (b) 0

(c) $-b$ (d) $b-a$

10. The value of $\sqrt{\frac{1}{2}}$ is

(a) π (b) $\frac{\pi}{2}$

(c) $\sqrt{\pi}$ (d) none

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) If P^* is a refinement of P then prove that $L(P,f,\alpha)\le L(P^*,f,\alpha)$.

Or

(b) If f is continuous on $[a,b]$ then prove that $f\in R(\alpha)$ on $[a,b]$.

12. (a) State and prove Cauchy criterion for uniform convergence.

Or

(b) Give an example to show that the convergent series of continuous function may have a discontinuous sum.

13. (a) Let α be monotonically increasing on $[a,b]$. Suppose $f_n\in R(\alpha)$ on $[a,b]$ for $n=1,2,\ldots$ and suppose $f_n\to f$ uniformly on $[a,b]$. Then prove that $f\in R(\alpha)$ on $[a,b]$ and $\int_a^b f\,d\alpha=\lim_{n\to\infty}\int_a^b f_n dx$.

Or

(b) If K is a compact metric space, if $f_n\in\mathscr{C}(K)$ for $n=1,2,\ldots$ and if $\{f_n\}$ converges uniformly on K, then prove that $\{f_n\}$ is equicontinuous on K.

[P.T.O.]

14. (a) Let $\mathscr{B}$ be the uniform closure of an algebraic $\mathscr{A}$ of bounded functions. Then prove that $\mathscr{B}$ is a uniformly closed algebra.

Or

(b) Suppose ΣC_n converges put $f(x)=\sum_{n=0}^{\infty} C_n x^n$ $(-1<x<1)$. Then prove that $\lim_{x\to 1} f(x)=\sum_{n=0}^{\infty} C_n$.

15. (a) State and prove Bessels inequality.

Or

(b) If f is a positive function on $(0, \infty)$ such that (i) $f(x+1)=xf(x)$ (ii) $f(1)=1$ (iii) $\log f$ is convex, then prove that $f(x)=\lceil x$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Prove that $f\in R(\alpha)$ on $[a,b]$ if and only if for every $\in>0$ there exists a partition P such that $U(P,f,\alpha)-L(P,f,\alpha)<\in$.

Or

(b) Assume α increases monotonically and $\alpha'\in \text{Re}$ on $[a,b]$. Let f be a bounded real function on $[a,b]$. Then prove that $f\in \text{Re}(\alpha)$ if and only if $f\alpha'\in \text{Re}$. In that case $\int_a^b f\,d\alpha=\int_a^b f(x)\,\alpha'(x)\,dx$.

17. (a) If γ' is continuous on $[a,b]$ then prove that γ' is rectifiable and $\wedge(\gamma)=\int_a^b |\gamma'(t)|\,dt$.

Or

(b) If $\{f_n\}$ is a sequence of continuous functions on E and if $f_n \in f$ uniformly on E. Then prove that f is continuous on E. Is converse true?

18. (a) Suppose $\{f_n\}$ is a sequence of functions differentiable on $[a,b]$ and such that $\{f_n(x_0)\}$ converges for some x_0 on $[a,b]$. If $\{f_n'\}$ converges uniformly on $[a,b]$ then prove that $\{f_n\}$ converges uniformly on $[a,b]$ to a function f and $f'(x)=\lim_{n\to\infty} f_n'(x)$ $(a\le x\le b)$.

Or

(b) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

19. (a) State and prove Stone-Weierstrass theorem.

Or

(b) Let the series $\sum C_n x^n$ converges for $|x| < R$ and define $f(x) = \sum_{n=0}^{\infty} C_n x^n$ $(|x| < R)$. Then prove that the given series converges uniformly on $[-R+E, R-E]$. Also prove that the function f is continuous and differentiable in $(-R, R)$ and $f'(x) = \Sigma n C_n x^{n-1}$.

20. (a) State and prove Stirling's formula.

Or

(b) Put $f(x) = x$ if $0 \le x \le 2\pi$ and apply Parseval's theorem to conclude that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

(8 pages) **Reg. No. :** ....

Code No. : 7835 **Sub. Code : PMAM 15**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

First Semester

Mathematics – Core

NUMERICAL ANALYSIS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $f(x)=\frac{1}{x^2}$ whose arguments are a, b, c then the first divided differences is ————.

(a) $\frac{-(a+b)}{a^2b^2}$ (b) $\frac{ab+bc+ca}{a^2b^2c^2}$

(c) $\frac{-1}{abcd}$ (d) $\frac{ab+bc}{a^2b^2c^2}$

2. Striling's formula give the most accurate result for the values of P lying between ————.

(a) $\frac{1}{4}\le P\le\frac{3}{4}$ (b) $0<P<1$

(c) $\frac{-1}{4}\le P\le\frac{1}{4}$ (d) $-1<P<0$

3. $\left(\frac{dy}{dx}\right)_{x=x_n}$ = ————.

(a) $\frac{1}{h}\left[\nabla y_n+\nabla^2 y_n+\nabla^3 y_n+.....\right]$

(b) $\frac{1}{h}\left[\nabla^2 y_n+\nabla^3 y_n+\frac{1}{3}\nabla^4 y_n+.....\right]$

(c) $\frac{1}{h}\left[\nabla y_n+\frac{1}{2}\nabla^2 y_n+\frac{1}{3}\nabla^3 y_n+.....\right]$

(d) $\frac{1}{h}\left[\nabla y_n+\frac{1}{2!}\nabla^2 y_n+\frac{1}{3!}\nabla^3 y_n+.....\right]$

4. $\left(\frac{d^2y}{dx^2}\right)_{x=x_0}$ = ————.

(a) $\frac{1}{h}\left[\Delta^2 y_0-\Delta^3 y_0+\frac{11}{12}\Delta^4 y_0-......\right]$

(b) $\frac{1}{h^2}\left[\frac{\Delta^2 y_0}{2!}-\frac{\Delta^3 y_0}{3!}+\frac{11}{12}\Delta^4 y_0-......\right]$

(c) $\frac{1}{h^2}\left[\Delta^2 y_0+\Delta^3 y_0+\frac{11}{12}\Delta^4 y_0+......\right]$

(d) $\frac{1}{h^2}\left[\Delta^2 y_0-\Delta^3 y_0+\frac{11}{12}\Delta^4 y_0+......\right]$

5. The value of $\int_0^6 \frac{dx}{1+x}$ using Weddley's rule is ________.

(a) 1.95285 (b) 2.95285

(c) 0.95285 (d) 1.75285

6. If $I_1 = 0.775$ and $I_2 = 0.7828$ then by using Romberg's formula for I_1 and $I_2 =$ ________.

(a) 0.7854 (b) 0.7825

(c) 0.6825 (d) 0.7835

7. $\frac{dy}{dx} = 1 - y$, $y(0) = 0$ then $y(0.1)$ is ________.

(a) 0.1 (b) 0.2

(c) 1 (d) 0

8. ________ method is the Runge – Kutta method of second order.

(a) Euler's method (b) Picard's method

(c) Modified Euler's (d) Taylor's method

9. Adam's corrector formula is ________.

(a) $y_{n+1}, C = y_n + \frac{h}{2!}\left[9y'_{n+1} + 19y'_n - 5y'_{n+1} + y'_{n-2}\right]$

(b) $y_{n+1}, C = y_n + \frac{h}{24}\left[9y'_{n+1} + 19y'_n - 5y'_{n+1} - y'_{n-2}\right]$

(c) $y_{n+1}, C = y_n + \frac{h}{24}\left[9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}\right]$

(d) $y_{n+1}, C = y_n + \frac{h}{2!}\left[9y'_{n+1} + 19y'_n - 5y'_{n+1} - y'_{n-2}\right]$

10. For solving differential equation in Milne's predictor formula the required number of initial values are ________.

(a) 4 (b) 3

(c) 2 (d) 1

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find a cubic polynomial which takes the following values.

x	0	1	2	3
$f(x)$	1	2	1	10

Or

(b) Find the equation of the cubic curve which passes through the points (4, –43), (7, 83), (9, 327) and (12, 1053). Hence find $f(10)$.

12. (a) Find $\frac{dy}{dx}$ at $x = 51$ from the following data

x	50	60	70	80	90
y	19.96	36.65	58.81	77.21	94.61

Or

(b) Give $u_0 = 5$, $u_1 = 15, u_2 = 57$ and $\frac{du}{dx} = 4a + x = 0$ and 72 at $x = 2$ find $\Delta^3 u_0$ and $\Delta^4 u_0$.

13. (a) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule with $h = 0.2$. Hence determine the value of π.

Or

(b) Calculate $\int_{0.5}^{0.7} e^{-x} x^{\frac{1}{2}} dx$ taking 5 ordinates by Simpson's $\frac{1}{3}$ rule.

14. (a) Using Taylor's method, find $y(0.1)$ correct to 3 decimal places from $\frac{dy}{dx} + 2xy = 1$, $y_0 = 0$.

Or

(b) Using Picard's method solve $\frac{dy}{dx} = 1 + xy$ with $y(0) = 2$. Find $y(0.1)$ and $y(0.2)$.

15. (a) Using Milne's predictor corrector method find $y(0.4)$ for the differential equation $\frac{dy}{dx} = 1 + xy, y(0) = 2$.

Or

(b) Using Adam's Bashforth method find $y(0.4)$ for the differential equation $y' = 1 + xy$, $y(0) = 2$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Using Striling's formula compute y_{35} given that $y_{10} = 600$; $y_{20} = 512$; $y_{30} = 439$; $y_{40} = 346$; $y_{50} = 243$.

Or

(b) Tabulate $y = x^3$ for $x = 2,3,4,5$ and calculate the cube root of 10 correct to three decimal places.

17. (a) Find $y'(x)$ given

x	0	1	2	3	4
$y(x)$	1	1	15	40	85

Hence find $y'(x)$ at $x = 0.5$.

Or

(b) A rod is rotating in a plane. The following table gives the angle θ (radians) through which the rod has turned for various values of time t (seconds).

t	0	0.2	0.4	0.6	0.8	1.0
θ	0	0.12	0.49	1.12	2.02	3.20

Calculate the angular velocity and the angular acceleration of the rod when $t = 0.6$ seconds.

18. (a) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Romberg's method correct to 4 decimal places. Hence deduce an approximate value of π.

Or

(b) Evaluate $I = \int_1^2 \int_1^2 \left(\frac{1}{x+y}\right) dx\, dy$ using Trapezoidal rule with $h = K = 0.25$.

19. (a) Given $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$, $y(1) = 1$, Evaluate $y(1.3)$ by modified Euler's method.

Or

(b) Compute $y(0.1)$ and $y(0.2)$ by Runge–Kutta method of 4th order for the differential equation $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$.

20. (a) Given $\frac{dy}{dx} = \frac{1}{x+y}$; $y(0) = 2$. If $y(0.2) = 2.09$, $y(0.4) = 2.17$ and $y(0.6) = 2.24$ find $y(0.8)$ using Milne's method.

Or

(b) Using Adams – Bashforth method, determine $y(1.4)$ given that $y' - x^2 y = x^2$, $y(1) = 1$. Obtain the starting values from Euler's method.

(6 pages) **Reg. No. :** ....

Code No. : 7842 **Sub. Code : PMAE 22**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Second Semester

Mathematics

Elective – DISCRETE MATHEMATICS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. In the conditional statement $p \to q, p$ is called ——————

(a) conjunction (b) inverse

(c) hypothesis (d) conclusion

2. Let $P(x)$ be the statement "$x = \frac{x}{2}$". Which of the following truth value is TRUE?

(a) $\exists x\, P(x)$ (b) $\forall x\, P(x)$

(c) $P(1)$ (d) $P(-1)$

3. How many three digit numbers can be formed from $\{1, 2, 3, 4, 5\}$?

(a) 15 (b) 125

(c) 120 (d) 243

4. In a company of 125 labours, 90 of them know Hindi, 25 of them know Malayalam and 10 of them know both. How many of these labours don't know both languages?

(a) 30 (b) 25

(c) 20 (d) 15

5. The symmetric closure of the relation $R = \{(a, b) : a > b\}$ is ————.

(a) $\{(a, b) : a \neq b\}$ (b) $\{[a, b] : a \neq b\}$

(c) $\{[a, b) : a > b\}$ (d) $\{(a, b] : a > b\}$

6. The transitivity closure of a relation R equals the ————.

(a) reflexive closure (b) connectivity relation

(c) symmetric closure (d) none of the above

7. The value of $(1+0)$ is ————.

(a) 1 (b) 0

(c) -1 (d) -1 or $+1$

8. In a Boolean Algegbra, the value of $x \wedge 1$ is ————.

(a) 1 (b) 0

(c) x (d) $\overline{x}$

9. A K – map for Boolean function of three variables consist of ———— cells.

(a) 2 (b) 4

(c) 6 (d) 8

10. If x, y are two input of AND gate, then the output is ————.

(a) $\overline{xy}$ (b) xy

(c) $\overline{x} + \overline{y}$ (d) $x + y$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Construct the truth table of $(p \vee \neg q) \rightarrow (p \wedge q)$

Or

(b) What are the negation of the following statements:

(i) All Indians like cake.

(ii) There is a bad boy in the class

12. (a) How many one-one functions are there from $\{1, 2, 3, 4\}$ to $\{a, b, c, d, e\}$?

Or

(b) Show that $C(n, r) = C(n, n-r)$, where n, r non-negative integers such that $r \leq n$.

13. (a) Let $A = \{1, 2, 3, 4, 5\}$ and R be the relation given by aRb if a divides b. Write R as a subset of $A \times A$

Or

(b) Let $M_{R_1} := \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ and $\mathrm{M}_{\mathrm{R}_2} := \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$:

Compute $M_{R_1} \vee M_{R_2}$ and $\mathrm{M}_{\mathrm{R}_1} \wedge M_{R_2}$.

14. (a) What do you mean by a Boolean Algebra B?

Or

(b) Find the duals of $x(y+0)$ and $\overline{x} \cdot 1 + (\overline{y} + z)$.

15. (a) Find the K– map for $x\bar{y}+\bar{x}y+\overline{xy}$.

Or

(b) Explain about the three Basic Types of Gates with examples.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) Show that $\neg(p \vee (\neg p \wedge \neg q))$ and $(\neg p \wedge \neg q)$ are logically equivalent.

Or

(b) Show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.

17. (a) Each user on a computer has a password, which is three to five character long, where each character is an uppercase letter or digit. Each password must contain at least one digit. How many possible password are there?

Or

(b) Show that the number of different subsets of a finite set S is $2^{|S|}$.

18. (a) What is the relation R on the set $\{1, 2, 3\}$ represented by the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Is R is reflexive, symmetric and or anti-symmetric?

Or

(b) Let R be a relation on a set A. Show that there is path length n, where n is a +ve integer, from a to $b \Leftrightarrow (a, b) \in R^n$.

19. (a) Find the values of the Boolean function represented by $F(x, y, z) = xy + \overline{z}$.

Or

(b) What do you mean by complementation, boolean sum, boolean product in the Boolean Algebra.

20. (a) Construct the circuit that produce the output $\overline{x}\left(\overline{y + z}\right)$.

Or

(b) Using Quine-McCluskey method, find the minimal expansion equivalent to $xyz + x\overline{y}z + \overline{x}yz + \overline{xy}z + \overline{xyz}$.

(6 pages) **Reg. No. : ....**

Code No. : 7114 **Sub. Code : PMAM 13**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

First Semester

Mathematics – Core

ANALYTIC NUMBER THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer:

1. $(ac, bc) = \underline{X}(a, b)$ where $X =$

   (a) c (b) $|c|$

   (c) ± 1 (d) $-c$

2. If $(a,b)=1$, $c\mid a$, $d\mid b$ the n $(c,d)=$

(a) 1 or 2

(b) ± 1

(c) 1

(d) 0

3. $\mu(8)=$

(a) 0

(b) 1

(c) -1

(d) 3

4. $\phi(10)=$

(a) 1

(b) 2

(c) 3

(d) 4

5. If f is multiplicative then $f(1)=$

(a) 0

(b) 1

(c) ± 1

(d) 2

6. $\overset{-1}{\lambda}(n)=$

(a) 1

(b) $|\mu(n)|$

(c) -1

(d) ± 1

7. A lattice point is a point with ———————— coordinates.

(a) zero

(b) rational

(c) integer

(d) irrational

8. $N(r)=$

(a) $2r+1$

(b) $2r^2+O(r)$

(c) $4r^2+O(r$

(d) $2[r]$

9. $\sum_{n=1}^{\infty}\frac{\mu(n)}{n^2}=$

(a) $\xi(2)$

(b) $\frac{6}{\pi^2}$

(c) $\frac{\pi^2}{6}$

(d) $\frac{\pi}{6}$

10. The smallest integer $x \geq 0$ for which x^2+x+41 is composite is

(a) 39

(b) 40

(c) 41

(d) 42

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that every integer $n>1$ is either a prime number or a product of prime numbers

Or

(b) If $(a,b)=1$, prove that $\left(a+b,a^2-ab+b^2\right)$ is either 1 or 3.

12. (a) If $n \geq 1$, prove that $\sum_{d/n} \phi(d) = n$.

Or

(b) Prove that $\phi(mn) = \phi(m)\phi(n)\frac{d}{\phi(d)}$ where $d = (m, n)$.

13. (a) Show that the möbius function is multiplicative but not completely multiplicative.

Or

(b) Prove that, for $n \geq 1$, $\sigma_\alpha^{-1}(n) = \sum_{d/n} d^\alpha \mu(d) \mu\left(\frac{n}{d}\right)$.

14. (a) If $x \geq 2$, prove that

$\sum_{n \leq x} \frac{d(n)}{n} = \frac{\log^2 x}{2} + 2C \log x + O(1)$ where C is Euler's constant.

Or

(b) If $x \geq 1$, $\alpha > 0$, $\alpha \neq 1$, show that $\sum_{n \leq x} \sigma_\alpha(n) = \frac{\xi(\alpha+1)}{\alpha+1} x^{\alpha+1} + O(x^B)$ where $\beta = \max\{1, \alpha\}$.

15. (a) State and prove Lagandre's identity.

Or

(b) State and prove Abel's identity.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove fundamental theorem of Arithmetic.

Or

(b) State and prove the division algorithm.

17. (a) For $n \geq 1$, prove that $\phi(n) = n \prod_{p/n}\left(1 - \frac{1}{p}\right)$.

Or

(b) If $n \geq 1$; show that

$$\wedge(n) = \sum_{d/n} \mu(d) \log \frac{n}{d} = -\sum_{d/n} \mu(d) \log d$$

18. (a) State and prove Euler's summation formula.

Or

(b) Prove that the set of lattice points visible from origin has density $\frac{6}{\pi^2}$.

19. (a) For $x \geq 1$, show that

$$\sum_{n \leq x} d(n) = x \log x + (2C - 1)x + O(\sqrt{x})$$ where C is Euler's constant.

Or

(b) Prove that, if both g and $f \times g$ are multiplicative then f is also multiplicative.

20. (a) If p_n denotes the n^{th} prime, prove that the following are logically equivalent:

$$\lim_{x \to \infty} \frac{\pi(x) \log x}{x} = 1 \text{ ————— (I)}$$

$$\lim_{x \to \infty} \frac{\pi(x) \log \pi(x)}{x} = 1 \text{ ————— (II)}$$

$$\lim_{x \to \infty} \frac{p_n}{n \log n} = 1 \text{ ————— (III)}$$

Or

(b) Prove that for every integer $n \geq 2$, $\frac{n}{6 \log n} < \pi(n) < \frac{6n}{\log n}$.

———————

(6 pages) **Reg. No. :** ....

Code No. : 7120 **Sub. Code : PMAM 24**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Second Semester

Mathematics – Core

DIFFERENTIAL GEOMETRY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer:

1. $[r', r'', r''']=$ ____________.

(a) k^2 (b) $k^2\tau$

(c) $k\tau$ (d) $k^3\tau$

2. The line of intersection of the normal plane and the osculating plane at P is the

(a) Normal plane (b) Tangent plane

(c) Principal plane (d) Principal normal

3. The pitch of the helix is ____________.

(a) $2\pi a$ (b) $2\pi b$

(c) ab (d) πb

4. If a curve lies on a sphere, then $\frac{d}{ds}(\sigma p')+\frac{p}{\sigma}=$ ____________.

(a) 1 (b) σ

(c) p (d) 0

5. ____________ point is defined as one for which $r_1 \times r_2 \neq 0$.

(a) Ordinary (b) Singular

(c) Osculating (d) Non-ordinary

6. The normal component of a is given by $a_n =$ ____________.

(a) $a \cdot b$ (b) $a \times b$

(c) $a \cdot N$ (d) $a \times N$

7. The two families are orthogonal if an only if ____________.

(a) $ER - FQ + GP = 0$

(b) $ER - 2FQ + GP = 0$

(c) $EP - 2FQ + GR = 0$

(d) $ER + 2FQ + GP = 0$

8. A necessary and sufficient condition for a Geodesic is ————.

(a) $U\frac{\partial T}{\partial \dot{v}} - V\frac{\partial T}{\partial \dot{u}} = 0$ (b) $U\frac{\partial T}{\partial \dot{v}} - V\frac{\partial T}{\partial \dot{u}} \neq 0$

(c) $U\frac{\partial T}{\partial \dot{v}} + V\frac{\partial T}{\partial \dot{u}} = 0$ (d) $U\frac{\partial T}{\partial \dot{v}} + V\frac{\partial T}{\partial \dot{u}} \neq 0$

9. The geodesic curvature of a Geodesic is ————.

(a) 1 (b) 0

(c) H (d) E

10. A point where $\frac{L}{E} = \frac{M}{F} = \frac{N}{G}$ is called ————.

(a) An ordinary (b) An essential

(c) An unbilic (d) A singularity

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Calculate the curvature and torsion of the cubic curve given by $r = (u, u^2, u_3)$.

Or

(b) State and prove Serret – Frenet formulae.

12. (a) Explain the oscillating circle.

Or

(b) Explain the oscillating sphere.

13. (a) For the paraboloid, $x = u$, $y = v$, $z = u^2 - v^2$, compute the values of E, F, G and H.

Or

(b) Discuss about the general helicoids.

14. (a) On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes $z =$ constant.

Or

(b) A helicoids generated by the screw motion of a straight line which meets the axis at an angle α. Find the orthogonal trajectories of the generators.

15. (a) Prove that if the orthogonal trajectories of the curve $v =$ constant are geodesics, then H^2 / E is independent of u.

Or

(b) Derive the formula

$$K_g = \frac{1}{H_{\dot{S}^3}}\left(\frac{\partial T}{\partial \dot{u}} V(t) - \frac{\partial T}{\partial \dot{v}} U(t)\right).$$

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) For the two quadratic surfaces, $ax^2 + by^2 + cz^2 = 1$ and $a'x^2 + b'y^2 + c'z^2 = 1$, Obtain the curvature and torsion of the curve of intersection.

Or

(b) Define (i) a curve at class in E_3. (ii) Binormal line at P. (iii) Change of parameter. (iv) Principal normal.

17. (a) Discuss the cylindrical helix.

Or

(b) Discuss about the circular helix.

18. (a) Find the coefficient of the direction which makes an angle $\pi/2$ with the direction whose coefficients are (l, m).

Or

(b) Define (i) anchor ring (ii) representation (iii) Circular helix. (iv) Singularity.

19. (a) Prove that on the curves of the family $v^3/u^2 =$ constant are geodesics on a surface with metric $v^2du^2 = 2uvdudv + 2u^2du^2$, $(u > 0, u > 0)$.

Or

(b) Prove that on the general surface a necessary and sufficient condition that the curve $v = c$ be a geodesic is $EE_2 + FE_1 - 2EF_1 = 0$.

20. (a) State and prove Liouvell's formula for K_g.

Or

(b) Derive the Rodrigue's formula for the lines of curvature.

————

(6 pages) **Reg. No. :** ....

Code No. : 7840 **Sub. Code : PMAM 25**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Second Semester

Mathematics – Core

GRAPH THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. If G is loopless and has exactly one spanning tree T, then

(a) $T \subset G$

(b) $T = G$

(c) $G \subset T$

(d) $T \neq G$

2. If G is either trivial or disconnected, then $K'(G) =$

(a) 0 (b) 1

(c) 2 (d) 3

3. Every k-critical 2-connected graph has a vector of degree

(a) 1 (b) 4

(c) 2 (d) 8

4. The number of edges of a graph $C_{2,5}$ is

(a) 2 (b) 5

(c) 7 (d) 4

5. The number of distinct perfect matching of K_4 is

(a) 1 (b) 2

(c) 3 (d) 4

6. The edge chromatic number of K_5 is

(a) 2 (b) 3

(c) 4 (d) 5

7. The Grotzsch graph is

(a) 1-critical (b) 2-critical

(c) 3-critical (d) 4-critical

8. The chromatic number of $K_{n,n}$ is

(a) 2 (b) 3

(c) n (d) $2n$

9. $K^4 - 3K^3 + 3K^2$ is the chromatic polynomial of

(a) K_4 (b) C_4

(c) P_4 (d) No graph

10. If G is simple graph, then for any edge e of G, $\pi_k(G) + \pi_k(G, e) =$

(a) $\pi_k(G+e)$ (b) $\pi_k(G-e)$

(c) $\pi_k(G)-1$ (d) $\pi_k(G)-e$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that G is a forest if and only if $\delta = \gamma - w$.

Or

(b) Prove that an edge e of G is a out edge of G if and only if e is contained in no cycle of G.

12. (a) Prove that the closure of G is well defined.

Or

(b) State and prove Dirac's theorem.

13. (a) Prove that every 3-regular graph without cut edges has a perfect matching.

Or

(b) State and prove Hall's theorem.

14. (a) Prove that in a bipartite graph, number of vertices in maximum independent set is equal to the number of edges in minimum covering.

Or

(b) Prove that :

(i) $r(3, 3) = 6$

(ii) $r(k, l) \leq \binom{k+l-2}{k-1}$.

15. (a) For any graph G, prove that $\chi(G) \leq \Delta + 1$.

Or

(b) In a critical graph, prove that no vertex cut is a clique.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that $H_{m,n}$ is m-connected.

Or

(b) Prove that the spanning tree obtained by Kruskal's algorithm is an optimal tree.

17. (a) Prove that a non-empty connected graph is Eulerian iff it has no vertices of odd degree.

Or

(b) Prove that a connected simple graph G with $\gamma \geq 3$ is Hamiltonian iff its closure $C(G)$ is Hamiltonian.

18. (a) State and prove Tute's theorem.

Or

(b) Prove that if G is simple then either $\chi' = \Delta$ (or) $\chi' = \Delta + 1$.

19. (a) If a simple graph G has not K_{m+1}, prove that G is degree majorized by some complete m-partite graph H. Also, if G has the same degree sequence as H, prove that $G \sim H$.

Or

(b) State and prove Turan's theorem.

20. (a) If G is simple, show that the coefficient of K^{8-1} in $\pi_K(G)$ is $-\varepsilon$.

Or

(b) For any positive integer K, prove that there exists a K-chromatic graph not containing a triangle.

———————

Reg. No. : ....

Code No. : 7837 **Sub. Code : PMAM 22**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Second Semester

Mathematics — Core

ANALYSIS II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If P is a partition of $[a, b]$, then $\int_a^{-b} f\, dx =$

(a) $\inf_p U(p, f)$ (b) $\sup_p L(p, f)$

(c) $\inf_p L(p, f)$ (d) $suf_p U(p, f)$

2. The unit step function I is defined by $I(n)=$

(a) $\begin{cases} 0 & if\ x<0 \\ 1 & if\ x \geq 0 \end{cases}$ (b) $\begin{cases} 0 & if\ x \leq 0 \\ 1 & if\ x > 0 \end{cases}$

(c) $\begin{cases} 0 & if\ x > 0 \\ 1 & if\ x \leq 0 \end{cases}$ (d) $\begin{cases} 0 & if\ x \geq 0 \\ 1 & if\ x < 0 \end{cases}$

3. If r' is continuous on $[a, b]$, then

(a) $\wedge(\gamma) = 0$ (b) $\wedge(\gamma) < \infty$

(c) $\wedge(\gamma) = b$ (d) $\wedge(\gamma) = a$

4. If X is a metric space, then $\mathcal{C}_b(x)$ will denote the set of all

(a) Complex valued continuous functions with domain X

(b) Real valued continuous functions with domain X

(c) Complex value continuous, bounded functions with domain X

(d) Real valued continuous, bounded functions with domain X

5. Which of the following statement is true?

(a) Every uniformly bounded sequence of continuous functions on $[0, 1]$ has a subsequence converging point wise on $[0, 1]$

(b) Every convergent sequence contains a uniformly convergent subsequence

(c) Every convergent sequence of uniformly bounded functions has a uniformly convergent subsequence

(d) Every uniformly convergent sequence of bounded functions is uniformly bounded

6. Let $f_n(x) = \dfrac{x^2}{x^2 + (1 - nx)^2}, 0 \leq x \leq 1, n = 1, 2, 3 \ldots$ Then $\{f_n\}$ is

(a) Uniformly bounded but not equicontinuous

(b) Equicontinuous but not uniformly bounded

(c) Uniformly bounded and equicontinuous

(d) Neither uniformly bounded nor equicontinuous

7. If $f(x)=\begin{cases} e^{-1/x^2} & if\ n \neq 0 \\ 0 & if\ x=0 \end{cases}$, then $f^{(3)}(0)$ is

(a) 1

(b) 0

(c) ∞

(d) $-\infty$

8. $\lim_{n \to 0} (1+x)^{1/x}$

(a) 0

(b) 1

(c) e

(d) ∞

9. With usual notation, the period of the function E is

(a) 2π

(b) $2\pi i$

(c) π

(d) πi

10. If $f(x)=0$ for all x in some segment J, then for $x \in J$ $\lim_{N \to \infty} x_N (f:x) =$

(a) x

(b) ∞

(c) 0

(d) 1

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Prove that $\int_{-a}^{b} f dx \leq \int_{a}^{-b} f \, dx$.

Or

(b) If $f \in R(\alpha)$ and $g \in R(\alpha)$ on $[a, b]$, then prove the following

(i) $f g \in R(\alpha)$

(ii) $|f| \in R(\alpha)$.

12. (a) Suppose K is compact and

(i) $\{f_n\}$ is a sequence of continuous functions on k.

(ii) $\{f_n\}$ converges point wise to a continuous function $\mathcal{C}$ on k.

(iii) $f_n(x) \geq f_{n+1}(x)$ for all $x \in k, n = 1, 2, 3 \ldots$ Then show that $f_n \to f$ uniformly on k.

Or

(b) Prove that the metric space $\mathcal{C}(x)$ is a complete metric space.

13. (a) Let x be monotonically increasing on $[a, b]$. Suppose $f_n \in \mathcal{R}(\alpha)$ on $[a, b]$ for $n=1,2,3...$, and suppose $f_n \to f$ uniformly on $[a, b]$. Prove that $f \in \mathcal{R}(\alpha)$ on $[a, b]$ and $\int_a^b f\, d\alpha = \lim_{n\to\infty} \int_a^b f_n\, d\alpha$.

Or

(b) Test the convergence of

$f_n(x) = \sin nx,\ 0 \le x \le 2\pi,\ n=1,2,3...$

14. (a) Suppose A is an algebra of functions on a set E, A separates points on E, and A vanishes at no point of E. Suppose x_1, x_2 are distinct points of E, and c_1, c_2 are constants. Prove that A contains a function f such that $f(x_1) = c_1$ and $f(x_2) = c_2$.

Or

(b) Find the limit of the function $\lim_{n\to 0} \frac{x - \sin x}{\tan x - x}$.

15. (a) Suppose $a_1, a_2, \ldots a_n$ are complex numbers, $n \geq 1, a_n \neq 0$. $p(z) = \sum_0^n a_k z^k$. Prove that $p(z) = 0$ for some complex number z.

Or

(b) If for some x, there are constants $f > 0$ and $m < \infty$ such that $|f(x+t) - f(x)| \leq m|t|$ for all $t \in (-\delta, \delta)$, then prove that $\lim_{N \to 0} x_N(f : x) = f(x)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Prove that $f \in \mathbb{R}(\alpha)$ on $[a, b]$ if and only if for every $\varepsilon > 0$ there exists a partition p such that $U(p, f, \alpha) - L(p, f, \alpha) < \varepsilon$.

Or

(b) If $f_1 \in \mathbb{R}(\alpha)$ $f_z \in \mathbb{R}(\alpha)$ on $[a, b]$, then prove the following.

(i) $f_1 + f_2 \in \mathbb{R}(\alpha)$, $cf, \in \mathbb{R}(\alpha)$ for every constant C and

$$\int_a^b (f_1 + f_2) d\alpha = \int_a^b f_1 \, d\alpha + \int_a^b f_2 \, d\alpha,$$

$$\int_a^b cf_1 \, d\alpha = c\int_a^b f_1 \, d\alpha.$$

(ii) If $f_1(x) \le f_2(x)$ on $[a,b]$, then

$$\int_a^b f_1 \, d\alpha \le \int_a^b f_2 \, d\alpha.$$

17. (a) If γ' is continuous on $[a, b]$, then prove that γ is rectifiable and $\wedge(\gamma) = \int_a^b |\gamma'(t)| \, dt$.

Or

(b) Suppose $\lim_{n\to\infty} f_n(x) = f(x)$, $x \in E$. Put $M_n = \sup_{x\in E} |f_n(x) - f(x)|$. Prove that $f_n \to f$ uniformly on E if and only if $M_n \to 0$ as $n \to \infty$.

18. (a) If K is compact, $f_n \in \mathscr{C}(K)$, $n = 1, 2, ..$ and if $\{f_n\}$ is pointwise bounded and equicontinuous on K, then prove that $\{f_n\}$ is uniformly bounded on k and $\{f_n\}$ contains a uniformly convergent subsequence.

Or

(b) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

19. (a) State and prove stone Weierstrass theorem.

Or

(b) Suppose the series $\sum a_n x^n$ and $\sum b_n x^n$ converse in the segment $s = (-R, R)$. Let E be the set of all $x \in S$ at which $\sum a_n x^n = \sum b_n x^n$. If E has a limit point in S, prove that $a_n = b_n$ for $n = 0, 1, 2, ...$ Hence prove $\sum a_n x^n = \sum b_n x^n$ for all $x \in S$.

20. (a) State and prove Parseval's theorem.

Or

(b) Put $f(x)=x$ if $0 \le x < 2\pi$ and prove that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \pi^2/6$ by applying Parseval's theorem.

(8 pages) Reg. No. : ....

Code No. : 7125 **Sub. Code : PMAM 31**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Third Semester

Mathematics (Core)

MEASURE AND INTEGRATION

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. Let Q' be the set of rational numbers in the interval $[0, 1]$. Then $m^*(Q')$ is

   (a) 0 (b) 1

   (c) ∞ (d) 1/2

2. If A is a measurable set of finite outer measure that is contained in B, then the excision property is

   (a) $m^*(A \sim B) = m^*(B) - m^*(A)$

   (b) $m^*(B \sim A) = m^*(B) - m^*(A)$

   (c) $m^*(B \sim A) = m^*(A) - m^*(B)$

   (d) $m^*(A \sim B) = m^*(A) - m^*(B)$

3. $f^-(x)$ is defined by

   (a) $f^-(x) = \max\{f(x), 0\}$

   (b) $f^-(x) = \min\{f(x), 0\}$

   (c) $f^-(x) = \max\{-f(x), 0\}$

   (d) $f^-(x) = \max\{f(x), -f(x)\}$

4. $\bigcap_{k=1}^{\infty}\{x \in E \,/\, f(x) > c - 1/k\}$ is

   (a) $\{x \in E \,/\, f(x) > c\}$ (b) $\{x \in E \,/\, f(x) \geq c\}$

   (c) $\{x \in E \,/\, f(x) = c\}$ (d) $\{x \in E \,/\, f(x) = \infty\}$

5. If $\psi = \sum_{i=1}^{n} a_i \cdot \chi_{E_i}$ on E then $\int_E \psi$ is

(a) $\sum_{i=1}^{n} a_i$ (b) $\sum_{i=1}^{n} a_i m(E_i)$

(c) $\bigcup_{i=1}^{n} E_i$ (d) $\sum_{i=1}^{n} a_i E_i$

6. Let f be a non-negative measurable function on E and $\lambda > 0$. Chebychev's inequality states that

(a) $m\{x \in E \ / \ f(x) \geq \lambda) \geq \frac{1}{\lambda} \int_E f$

(b) $m\{x \in E \ / \ f(x) \geq \lambda) \leq \frac{1}{\lambda} \int_E f$

(c) $m\{x \in E \ / \ f(x) \geq \lambda) \leq \lambda \int_E f$

(d) $m\{x \in E \ / \ f(x) \leq \lambda) \leq \frac{1}{\lambda} \int_E f$

7. Which one of the following is not true?

(a) $f^+(x) = \max\{f(x), 0\}$

(b) $f^-(x) = \max\{-f(x), 0\}$

(c) $f = f^+ + f^-$

(d) $f^+ - |f| = -f^1$

8. Let f be a monotone function on (a, b) and $x_0 \in (a, b)$. Then $f(x_0^-)$ is

(a) $\sup\{f(x) \ / \ a < x < x_0\}$

(b) $\inf\{f(x) \ / \ a < x < x_0\}$

(c) $\sup\{f(x) \ / \ x_0 < x < b\}$

(d) $\inf\{f(x) \ / \ x_0 < x < b\}$

9. The function f defined on $[0, 1]$ by $f(x) = \sqrt{x}$, $0 \leq x \leq 1$ is

(a) absolutely continuous and Lipschitz

(b) absolutely continuous but not Lipschitz

(c) Lipschitz but not absolutely continuous

(d) Neither absolutely continuous nor Lipschitz

10. Which one of the following is not true?

(a) Lebesgue measure on $[0, 1]$ is a finite measure

(b) Lebesgue measure on $[-\infty, \infty]$ is a σ-finite measure

(c) Counting measure on an uncountable set is σ-finite

(d) Lebesgue measure on the real line is complete

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Prove that the union of finite collection of measurable sets is measurable.

Or

(b) Define a measurable set and prove that any set of outer measure zero is measurable.

12. (a) Let f and g be measurable functions on E that are finite a.e. on E. For any α and β, prove that $\alpha f + \beta g$ is measurable on E.

Or

(b) State and prove the simple approximation lemma.

13. (a) Give an example of a uniformly bounded sequence of Riemann integrable functions on a closed bounded interval which converges pointwise to a function that is not Riemann integrable.

Or

(b) Let f be a non-negative measurable function on E. Prove that $\int_E f = 0$ if and only if $f = 0$ a.e. on E.

14. (a) Let f be a measurable function on E. Prove that f^+ and f^- are integrable over E if and only if $|f|$ is integrable over E.

Or

(b) State and prove Jordan's theorem.

15. (a) Let the function f be absolutely continuous on $[a, b]$. Prove that f is differentiable almost everywhere on (a, b), its derivative f' is integrable over $[a, b]$ and $\int_a^b f' = f(b) - f(a)$.

Or

(b) Prove that the union of a countable collection of measurable sets is measurable in a general measure space.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Prove that every internal is measurable.

Or

(b) Prove that Lebesgue measure possesses the following continuity properties :

(i) If $\{A_k\}$ is an ascending collection of measurable sets then $m\left(\bigcup_1^\infty A_k\right) = \lim_{k\to\infty} m(A_k)$.

(ii) If $\{B_k\}$ is a descending collection of measurable sets and $m(B_1) < \infty$ then $m\left(\bigcap_1^\infty B_k\right) = \lim_{k\to\infty} m(B_k)$.

17. (a) Let $\{f_n\}$ be a sequence of measurable functions on E that converges pointwise a.e. on E to the function f. Prove that f is measurable.

Or

(b) State and prove Egoroff's theorem.

18. (a) State and prove the bounded convergence theorem.

Or

(b) State and prove Fatou's lemma.

19. (a) Let the functions f and g be integrable over E. For any α and β, prove that the function $\alpha f + \beta g$ is integrable over E and $\int_E (\alpha f + \beta g) = \alpha \int_E f + \beta \int_E g$. Also prove that $\int_E f \le \int_E g$ if $f \le g$ on E.

Or

(b) If the function on f is monotonic on the open interval (a, b), prove that it is differentiable almost everywhere on (a, b).

20. (a) State and prove Jordan Decomposition theorem.

Or

(b) Define a signed measure and state and prove Hahn's lemma.

(7 pages) **Reg. No. :** ....

Code No. : 7849 **Sub. Code : PMAE 31**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Third Semester

Mathematics

Elective – ALGEBRAIC NUMBER THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The equation $ax + by = c$ is equivalent to the congruence ————.

(a) $ax \equiv c(\operatorname{mod} b)$

(b) $ax \equiv b(\operatorname{mod} c)$

(c) $ax \equiv by(\operatorname{mod} c)$

(d) $ax \equiv cy(\operatorname{mod} b)$

2. All solutions of $3x + 5y = 1$ can be written in the form ————.

(a) $x = 2 + 5t,\ y = -1 - 3t$

(b) $x = -2 + 5t,\ y = -1 - 3t$

(c) $x = 3 + 5t,\ y = -2 - 3t$

(d) $x = 2 - 3t,\ y = -1 + 5t$

3. $x^2 + y^2 = z^4$ has ———— with $(x, y, z) = 1$.

(a) unique solution

(b) no solution

(c) infinitely many solutions

(d) exactly four solutions

4. A solution x_1, y_1, z_1 having the property that these three are relatively prime in pairs is called as ———— solution.

(a) prime (b) relatively prime

(c) primal (d) primitive

5. $K_n|K_{n-1}\xi - h_{n-1}| + K_{n-1}|K_n\xi - h_n| =$ ————.

(a) 0 (b) 1

(c) ξ (d) n

6. Which of the following is not irrational?

(a) π (b) e

(c) $\sqrt{2}+\sqrt{3}$ (d) none

7. $|\xi K_n - h_n| \geq \frac{1}{K_{n+1}}$ for ———— $n \geq 0$.

(a) no value of (b) any

(c) only one (d) some

8. The units of the rational number field Q are ————.

(a) 0, 1 (b) –1, 1

(c) 0, –1 (d) 0, 1, –1

9. If an integer α in $Q(\sqrt{m})$ is neither zero nor a unit, then ————.

(a) $|N(\alpha)| < 1$ (b) $|N(\alpha)| \leq 1$

(c) $|N(\alpha)| > 1$ (d) $|N(\alpha)| \geq 1$

10. A quadratic field $Q(\sqrt{m})$ is called real if ————.

(a) $m = 0$ (b) $m > 0$

(c) $m > 1$ (d) $m < 0$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $ax + by = a + c$ is solvable if and only if $ax + by = c$ is solvable

Or

(b) Find all solutions in positive integers of $5x + 3y = 52$.

12. (a) Show that $x^4 - y^4 = z^2$ has no solutions with $yz \neq 0$.

Or

(b) Prove that 4993 is a prime number.

13. (a) Show that two distinct infinite simple continued fractions converge to different values.

Or

(b) Expand $\sqrt{2}$ as an infinite simple continued fraction.

14. (a) Let ξ denote any irrational number. If there is a rational number a/b with $b \geq 1$ such that $\left|\xi - \frac{a}{b}\right| < \frac{1}{2b^2}$, then show that a/b equals one of the convergents of the simple continued fraction expansion of ξ.

Or

(b) Let $\alpha = \alpha_1 + \alpha_2 i$ be an algebraic number, where α_1 and α_2 are real. Does it follow that α_1 and α_2 necessarily be algebraic integers?

15. (a) If γ is an integer in $Q(\sqrt{m})$, then show that $N(\gamma) = \pm 1$ if and only if γ is a unit.

Or

(b) Prove that there are infinitely many units in any real quadratic fields.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve the equation $x + 2y + 3z = 1$.

Or

(b) Prove that $ax + by = c$ is solvable if and only if $(a, b) = (a, b, c)$.

17. (a) Show that the positive primitive solutions of $x^2 + y^2 = z^2$ with y even are $x = r^2 - s^2$, $y = 2rs$, $z = r^2 + s^2$, where r and s are arbitrary integers of opposite parity with $r > s > 0$ and $(r, s) = 1$.

Or

(b) Prove that the only integral solutions of $x^4 + y^4 = z^2$ are the trivial solutions $x = 0, y, z = \pm y^2$ and $x, y = 0$, $z = \pm x^2$.

18. (a) If $ax^2 + by^2 + cz^2 = 0$ has a solution in integers x, y, z not all zero, then prove that a, b, c do not have the same sign and that $-bc, -ac, -ab$ are quadratic residues modulo a, b, c respectively.

Or

(b) Prove that any irrational number ξ is uniquely expressible as an infinite simple continued fraction.

19. (a) If a/b is a rational number with positive denominator such that $|\xi - a/b| < |\xi - h_n/K_n|$ for some $n \geq 1$, then show that $b > K_n$. Also prove that if $|\xi b - a| < |\xi K_n - h_n|$ for some $n \geq 0$, then $b \geq K_{n+1}$

Or

(b) Prove :

(i) If α is any algebraic number, then there is a rational integer b such that $b\alpha$ is an algebraic integer.

(ii) The reciprocal of a unit is a unit.

(iii) The units of an algebraic number field from a multiplication group.

20. (a) Prove :

(i) Every quadratic field is of the form $Q(\sqrt{m})$ where M is a square-free rational integer, positive or negative but not equal to 1.

(ii) The units of $Q(\sqrt{2})$ are $\pm(1+\sqrt{2})^n$ where n ranges over all integers.

Or

(b) Let M be a negative square-free rational integer. Prove that the field $Q(\sqrt{m})$ has units ± 1 and these are the only units except in the cases $m=-1$ and $m=-3$. What will happen if $m=-1$ or $m=-3$?

(7 pages) **Reg. No.: ....**

Code No. : 7839 **Sub. Code : PMAM 24**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Second Semester

Mathematics — Core

DIFFERENTIAL GEOMETRY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer

1. The radius of curvature of the curve $= r = (a \cos u, b \sin u, a \cos 2u)$ at $u = \pi/4$ is

(a) 5a (b) a

(c) $5a/4$ (d) $a/4$

2. The equation of the rectifying plane is

(a) $(R-r).n=0$ (b) $(R-r).b=0$

(c) $(R-r).a=0$ (d) $(R-r).t=0$

3. The osculating circle has ———— point contact with the space curve.

(a) 2 (b) 4

(c) 3 (d) 1

4. The involutes of circular helin are

(a) Straight lines (b) Circles

(c) Plane curves (d) Spheres

5. If ω is one angle between the parametric curves at the point of intersection, then $\tan\omega$ is

(a) $H/\sqrt{EG}$ (b) $F/\sqrt{EG}$

(c) H/F (d) F/H

6. The surfaces generated by the screw motion of the $x-axis$ about the z axis is called a

(a) anchor ring (b) right helicoid

(c) general helicoid (d) paraboloid

7. The orthogonal trajectories of the circles $r = a\cos\theta$ is

(a) $r = a\sin\theta$ (b) $r = a\tan\theta$

(c) $r = a\sec\theta$ (d) $r = a\,\mathrm{cosec}\,\theta$

8. The number of different types of geodesics on a surface of revolution $r = (g(u)\cos v, g(u)\sin v, f(u))$ are

(a) 2 (b) 3

(c) 4 (d) 1

9. When polar developable of a space curve is a cone, the space curve lies on a

(a) Cylinder (b) Sphere

(c) Cone (d) Paraboloid

10. Any point on the paraboloid $r = (u\cos v, u\sin v, u^2)$ is

(a) Parabolic

(b) Elliptic

(c) Hyperbolic

(d) Either elliptic or parabolic

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the are length of one complete turn of the circular helix $r(u) = (a\cos u, a\sin u, bu)$, $-\alpha < u < \alpha$ where $a > 0$ and obtain the equation of the helix with s as parameter.

Or

(b) Find the Serrer–Frenet approximation of the curve $(\cos u, \sin u, u)$ at $u = \pi/2$.

12. (a) Prove that the condition of a surface having n point contact with the curve γ are invariant over a change of parameter.

Or

(b) Show that a necessary and sufficient condition that a curve lies on a sphere is $\frac{\rho}{\sigma} + \frac{d}{ds}(\sigma e') = 0$ at every point of the curve.

13. (a) Prove that the equation of a tangent plant at ρ on a surface with position vector $r = r(u,v)$ is either $R = r + ar_1 + br_2$ or $(R - r).(r_1 \times r_2) = 0$ where a and b are parameter.

Or

(b) Find the parametric directions and the angle between the parametric curves.

14. (a) Show that a curve on a sphere is geodesic if and only if it is a great circle.

Or

(b) If θ is the angle between the two curves given by the double family $pdu^2 + 2Qdudv + Rdv^2$ at a point (u,v) on the surfaces, then prove that

$$\tan\theta = \frac{2H\left(Q^2 - PR\right)^{1/2}}{ER - 2FQ + GP}.$$

15. (a) For any curve on a surface, prove that the geodesic curvature vector in intrinsic.

Or

(b) Prove that L,M,N vanish at all point of a surface iff the surface is the plant

PART C — $(5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) State and prove that existence theorem for finite upper bound for the set $L(\Delta)$ and given a formula for it.

Or

(b) Find the directions and equations of the tangent, normal and binomial and also obtain the normal, rectifying and osculating planes at a point on the circular helix $r = \left(a\cos\left(s/c\right), a\sin\left(s/c\right), \frac{bs}{c}\right)$.

17. (a) Find the curvature and torsion of the curves given by

(i) $y = f(x), z = g(x)$

(ii) $r = \left[a\left(3u - u^3\right), 3au^2, a\left(3u + u^3\right)\right]$

Or

(b) Prove that the radius of curvature e_1 of the locus of the centre of curvature is $\rho_1 = \left[\left\{\frac{\rho^2\sigma}{R^3}\frac{d}{ds}\left(\frac{\sigma\rho'}{\rho}\right) - \frac{1}{R}\right\}^2 + \frac{\rho'^2\sigma^4}{\rho^2R^4}\right]^{-1/2}$ where R is the radius of spherical curvature.

18. (a) The position vector of any point on the surface of revolution generated by the curve $[g(u), o, f(u)]$ in the XOZ plane is $r = [g(u)\cos v, g(u)\sin v, f(u)]$ where v is the angle of rotation about the z axis.

Or

(b) If (l', m') are direction coefficients of line which makes an angle $\pi/2$ with the line whose direction coefficients are (l, m) then prove that

$$l' = \frac{1}{4}(Fl + Gm), m' = \frac{1}{H}(El + Fm).$$

19. (a) Prove that any curve $u = u(t), v = v(t)$ on a surface $r = r(u, v)$ is a geodesic if and only if the principal normal at every point on the curve is normal to the surface.

Or

(b) State and prove Rodrigo's formula.

20. (a) prove that the condition for an elliptic, parabolic or hyperbolic points are independent of the particular parametric representation.

Or

(b) If U and V are the intrinsic quantities of a surface at a point (u, v), then prove that

$$kg = \frac{1}{H}\frac{V(s)}{u'} \text{ and } Kg = -\frac{1}{H}\frac{U(s)}{v'}$$

———————

(7 pages) **Reg. No. :** ....

Code No. : 7136 **Sub. Code : PMAM 44**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Fourth Semester

Mathematics — Core

TOPOLOGY — II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The space R_K (K topology) is

   (a) T_4

   (b) $T_{3\frac{1}{2}}$ but not T_4

   (c) T_3 but not $T_{3\frac{1}{2}}$

   (d) T_2 but not T_3

2. Regular space is also known as

   (a) T_4

   (b) $T_{2\frac{1}{2}}$

   (c) $T_{3\frac{1}{2}}$

   (d) T_3

3. Which one of the following is normal

   (a) R_l

   (b) R_l^2

   (c) $S_\Omega \times \overline{S_\Omega}$

   (d) R^J, J is uncountable

4. A space X is completely regular then it is homeomorphic to a subspace of

   (a) $[0,1]^J$

   (b) $\mathbb{R}^n$ where n is a finite

   (c) $\mathbb{R}^J$

   (d) $(0,1)^J$ where n is a finite number and J is uncountable

5. Tietze extension theorem implies

   (a) The Urysohn Metrization theorem

   (b) Heine-Borel Theorem

   (c) The Urysohn Lemma

   (d) The Tychonof Theorem

6. Indicate the correct answer

(a) Subspace of a Normal space is normal

(b) Product of Normal spaces is normal

(c) R_l^2 is completely regular

(d) R_K is regular but not normal

7. Which one of the following is locally finite in R?

(a) $\{(n-1, n+1) : n \in Z\}$

(b) $\left\{\left(0, \frac{1}{n} : n \in Z_+\right)\right\}$

(c) $\left\{\left(\frac{1}{n+1}, \frac{1}{n}\right) : n \in Z_+\right\}$

(d) $\{(x, x+1) : x \in R\}$

8. Let $\mathcal{A} = \{(n-1, n+1) : n \in Z\}$. Which of the following refine $\mathcal{A}$.

(a) $\left\{\left(n-\frac{1}{2}, n+\frac{3}{2}\right) : n \in Z_+\right\}$

(b) $\left\{n+\frac{1}{2}, n+\frac{3}{2}\right) : n \in Z_+\right\}$

(c) $\left\{\left(n-\frac{1}{2}, n+2\right) : n \in Z_+\right\}$

(d) $\{(x, x+1) : x \in R\}$

9. Which one of the following is false?

(a) Any set X with discrete topology is a Baire space

(b) The set of irrationals is not a Baire spaces

(c) [0, 1] is a Baire space

(d) Every locally compact space is a Baire space

10. Which of the following is not true?

(a) Every non empty open subset of the set of irrational numbers is of first category

(b) Open subspace of a Baire space is a Baire space

(c) Rationals as a subspace of real numbers is not a Baire space

(d) If $X = \bigcup_{n=1}^{x} B_n$ and X is a Baire space with $B_1 \neq \phi$, then atleast one of $\overline{B}_n$ has nonempty interior

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define $\mathbb{R}_K$ topological space. Prove that the space $\mathbb{R}_K$ is Hausdorff but not regular.

Or

(b) Prove that $\mathbb{R}_l^2$ is not a Lindeloff space.

12. (a) Show that the compact subset of a Hausdorff space is closed. Give an example of a closed subset of a Hausdorff space which is not compact.

Or

(b) Show that a compact Hausdorff space is normal.

13. (a) Prove that Tietze extension theorem implies the Urysohn Lemma.

Or

(b) State and prove imbedding theorem.

14. (a) Let $\mathcal{A}$ be a locally finite collection of subsets of X. Then prove that

(i) The collection $\mathcal{B}=\{\overline{A}:A\in\mathcal{A}\}$ is locally finite

(ii) $\overline{\bigcup_{A\in\mathcal{A}} A}=\bigcup_{A\in\mathcal{A}} \overline{A}$

Or

(b) Define finite intersection property. Let X be a set and D be the set of all subsets of X that is maximal with respect to finite intersection property. Show that :

(i) $x\in\overline{A}\ \forall A\in D$ if and only if every neighborhood of x belongs to D.

(ii) Let $A\in D$. Then prove that $B\supset A\Rightarrow B\in D$

15. (a) Define second category space. Prove that any open subspace Y of a Baire space X is also a Baire space.

Or

(b) Define a Baire space. Whether Q the set of rationals as a space is a Baire space? What about if we consider Q as a subspace of real numbers space. Justify your answer

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) What are the countability axioms. Prove that the space $\mathbb{R}_L$ satisfies all the countability axioms but the second.

Or

(b) Prove that a normal space is a regular space but not conversely.

17. (a) Define a regular space, normal space and a second countable space. Prove that every regular second countable space is normal.

Or

(b) (i) Prove that every normal space is completely regular and completely regular space is regular.

(ii) Prove that product of completely regular spaces is completely regular.

18. (a) State and prove Tietze extension theorem.

Or

(b) Prove that every regular second countable space is metrizable.

19. (a) Let X be a metrizable space. If A is an open covering of X, then prove that there is an open covering ξ of X refining A that is countably locally finite.

Or

(b) State and prove Tychonoff theorem.

20. (a) State and prove Baire Category theorem.

Or

(b) Let X be a space; let (Y,d) be a metric space. Let $f_n : X \to Y$ be a sequence of continuous functions such that $f_n(x) \to f(x)$ for all $x \in X$, where $f : X \to Y$. If X is a Baire space, prove that the set of points at which f is continuous is dense in X.

(8 pages) **Reg. No. :** ....

Code No. : 7844 **Sub. Code : PMAM 31**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2019.

Third Semester

Mathematics – Core

MEASURE AND INTEGRATION

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. E is measurable if

(a) If A is any set then $m^*(A) = m^*(A \cap E)$

(b) There exists a G_8 set $G \subset E$ such that $m^*(G \vee E) = 0$

(c) For each $\in > 0$ there exists a closed set $F \subset E$ for which $m^*(E \vee F) = 0$

(d) None of these

2. A countable set has outer measure ————.

(a) 0 (b) 1

(c) ∞ (d) finite

3. $\{x \in E / \mathfrak{S}(x) > c\} =$ ————.

(a) $\cup_{n=1}^{\infty} \{x \in E / \mathfrak{S}(x) \geq c + \frac{1}{n}\}$

(b) $\cap_{n=1}^{\infty} \{x \in E / \mathfrak{S}(x) \geq c + \frac{1}{n}\}$

(c) $\cup_{n=1}^{\infty} \{x \in E / \mathfrak{S}(x) > c + \frac{1}{n}\}$

(d) $\cap_{n=1}^{\infty} \{x \in E / \mathfrak{S}(x) > c + \frac{1}{n}\}$

4. Which one of the following is false?

(a) f is measurable if $\mathfrak{S}^{-1}(0)$ is measurable for any open set O or R

(b) A continuous real valued function on its measurable domain is measurable.

(c) A monotonic function defined on an interval is measurable.

(d) Composition of any two measurable functions is always measurable.

5. Let f be a bounded measurable function on E. Let then $E = \{\text{Rationals in } [0,1] \text{ only}\}$ let $f = 1.x_E$ on $[0,1]$.

(a) $\int_{[0,1]} f = 0$

(b) $\int_{[0,1]} f = 1$

(c) Integral does not exist on [0, 1]

(d) None of these

6. Let the non-negative function f be integrable over E. Then $\mathfrak{f}$ is ———— on E.

(a) finite $a-e$ (b) finite

(c) Zero (d) Constant

7. Let $\mathfrak{f}$ be monotonic function on (a,b). Then $\mathfrak{f}$ is continuous except possibly at

(a) Countable number of points in *(a, b)*

(b) Finite number of points in *(a, b)*

(c) Uncountable number of points in *(a, b)*

(d) None of the above

8. A closed interval *[c,d]* is said to be non-degenerate is

(a) $c > d$ (b) $c < d$

(c) $c = d$ (d) None

9. $\mathfrak{F}$ lip, $\mathfrak{F}$AC, $\mathfrak{F}$BV, denote the family of functions on [a, b] that are Lipschitz, absolutely continuous and bounded variation respectively. Then

(a) $\mathfrak{F}\text{lip} \subseteq \mathfrak{F}\text{AC} \subseteq \mathfrak{F}\text{BV}$

(b) $\mathfrak{F}\text{lip} \subseteq \mathfrak{F}\text{BV} \subseteq \mathfrak{F}\text{AC}$

(c) $\mathfrak{F}\text{AC} \subseteq \mathfrak{F}\text{BV} \subseteq \mathfrak{F}\text{lip}$

(d) $\mathfrak{F}\text{BV} \subseteq \mathfrak{F} \cap \text{C} \subseteq \mathfrak{F}\text{lip}$

10. The counting measure on an uncounatable set is ————

(a) σ-finite (b) not σ-finite

(c) σ-infinite (d) finite

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that for any bounded set E, there exists a G_8 set G for which $E \subset G$ and $m^*(E) = m^*(G)$.

Or

(b) Prove that the translate of a measurable set in measurable.

12. (a) Prove that a monotone function defined on an interval is measurable.

Or

(b) Let $\{fn\}$ be a sequence of measurable functions on E that converges pointwise almost every where on E to the functions f, Then show that f is measurable.

13. (a) Let E have measure zero. Let f be a bounded function on E. Then show that f is measurable and $\int_E f = 0$.

Or

(b) Let $\{fn\}$ be a sequence of bounded measurable functions on a set of finite measure E. If $\{fn\}$ converges to f uniformly on E, then show that $\lim_{n\to\infty} \int_E fn = \int_E f$

14. (a) Let f be integrable over E. Assume A and B are disjoint measurable subsets of E. Then show that $\int_{A\cup B} f = \int_A f + \int_B f$

Or

(b) Let f be an increasing function on the closed, bounded interval $[a, b]$. Then show that for each $\alpha > 0, m^*\{x \in (a,b) / \overline{D}\, f(x) \geq \alpha\} \leq \frac{1}{\alpha}[f(b) - f(a)]$ and $m^*\{x \in (a,b) / \overline{D}\, f(x) = \infty\} = 0$.

15. (a) Let the function f be absolutely continuous on the closed, bounded interval $[a/b]$. Then show that f is the difference of increasing absolute continuous functions and, in particular, f is of bounded variation.

Or

(b) Let γ be a signed measure on the measurable space $(X_1 \mathcal{M})$. Then show that every measurable subset of a positive set is itself a positive set and the union of countable collection of positive sets is positive.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that outer measure of intervals is its length.

Or

(b) Prove that the collection of lebesgue measurable sets form a σ-algebra.

17. (a) State and prove Lusin's theorem.

Or

(b) (i) If ℰ is an entended real values measurable function on E and $ℰ = g$ $a-e$ on E, then show that g is measurable on E.

(ii) If ℰ and g are measurable functions on E that are finite a-e on E then show that $af + \beta g$ is measurable on E for any α and β and also show that $ℰg$ is measurable on E.

18. (a) Let ℰ and g be bounded measurable functions on a set of finite measure E. Then show that for any α and β, $\int_E (\alpha ℰ + \beta g) = \alpha \int_E f + \beta \int_E g$.

More over, if $ℰ \le g$ on E, show that $\int_E f \le \int_E g$.

Or

(b) State and prove Bounded Convergence theorem.

19. (a) State and prove Vitali Covering Lemma.

Or

(b) (i) For $a \le u < v \le b$, show that $\int_a^b \text{Diff}_h\, f(x)\,dx = AV_h f(v) - AV_h f(u)$.

(ii) Let ℰ be an increasing function on closed bounded interval $[a, b]$. Then show that $ℰ'$ is integrable on $[a, b]$ and $\int_a^b f' \le f(b) - f(a)$.

20. (a) State and prove Hahn's Lemma and the Hahn decomposition theorem.

Or

(b) Prove the following:

(i) Let $\mathcal{S}$ be a collection of subsets of set X and $\mu : \mathcal{S} \to [0, \infty]$ a set function. Define $\mu^*(\phi) = 0$ and for $E \in \mathcal{S}$, $E \ne \phi$, define $\mu^*(E) = \inf \sum_{k=1}^{\infty} (\mu(Eu)$ where the infimum is taken over all countable collections $\{Eu\}_{n=1}^{\infty}$ of sets is $\mathcal{S}$ that cover E. The show that the set fucntion $\mu^* : 2^X \to [0, \infty]$ is an outer measure called the outer measure induced by μ.

(ii) Let $\mu : \mathcal{S} \to [0, \infty]$ be a set function defined on a collections of $\mathcal{S}$ of subsets of a set X and $\bar{\mu} : \mathcal{M} \to [0, \infty]$ the cartheodary measure induced by μ. Let $E \subset X$ for which $\mu^*(E) < \infty$. Then show that there exists a subset A of X for which $A \in \mathcal{S}_{\sigma\delta}$, $E \subseteq A$ and $\mu^*(E) = \mu^*(A)$. Furthermore, if E and each set in $\mathcal{S}$ is measurable with respect to μ^*, then so is A and $\bar{\mu}(AvE) = 0$.

(8 pages) Reg. No. : ....

Code No. : 7121 **Sub. Code : PMAM 25**

M.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2019.

Second Semester

Mathematics — Core

GRAPH THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. If G is a tree with υ vertices and ε edges then, $\sum_{\upsilon \in V} d(\upsilon) =$

   (a) $\upsilon + \varepsilon$

   (b) $\upsilon - \varepsilon + 1$

   (c) $\upsilon + \varepsilon + 1$

   (d) $\upsilon + \varepsilon - 1$

2. Which of the following is not true?

(a) A tree with more than 3 vertices has atleast two vertices which are cut vertices

(b) If the graph is acyclic then it is a forest

(c) A forest has ε cut edges

(d) A tree has ε cut edges

3. Which of the following is true?

(a) Every Eulerian graph is Hamiltonian

(b) Every Hamiltonian graph is Eulerian

(c) $K_{5,5}$ is both Eulerian and Hamiltonian

(d) If G is Hamiltonian, then G has no pendant vertex

4. The complete bipartite graph $K_{4,6}$ is

(a) both Eulerian and Hamiltonian

(b) Eulerian but not Hamiltonian

(c) Not Eulerian but Hamiltonian

(d) Neither Eulerian nor Hamiltonian

5. Which one of the following is true?

(a) $K_{2n,2n+2}; n \geq 2$ has a perfect matching

(b) Every 3-regular graph has a perfect matching

(c) A cycle on 12 vertices has a perfect matching

(d) Every graph on $2n$ vertices has a perfect matching

6. A complete graph on odd number of vertices has

(a) perfect matching

(b) no maximum matching

(c) maximum matching but no perfect matching

(d) can not say

7. Which of the following is not true?

(a) K_n has no independent set

(b) Every independent set is contained in a maximum independent set

(c) Covering number and the independence number are equal for trees with even number of vertices

(d) Covering number is always less than or equal to the independence number

8. The number of vertices in a minimum covering of a graph G is denoted by

(a) α' (b) α

(c) β' (d) β

9. If G is a simple graph

(a) $\pi_k(G-e)+\pi_k(G)=\pi_k(G\cdot e)$

(b) $\pi_k(G-e)+\pi_k(G\cdot e)=\pi_k(G)$

(c) $\pi_k(G\cdot e)+\pi_k(G)=\pi_k(G-e)$

(d) None of these

10. A ———————— graph has no vertex cut which is a clique.

(a) tree (b) chromatic

(c) critical (d) connected

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define a tree and show that a graph G is a tree if and only if any two vertices of G are connected by a unique path.

Or

(b) When a graph is said to be m-connected? Prove that a graph with more than 2 vertices is 2-connected if and only if any two vertices of G are connected by atleast two internally disjoint paths.

12. (a) Define a Hamiltonian graph. If G is a simple graph with $\upsilon \geq 3$ and $\delta \geq \frac{\upsilon}{2}$ then prove that G is Hamiltonian.

Or

(b) Define $c(G)$. Show that $c(G)$ is well defined.

13. (a) Define perfect matching. If G has a perfect matching then show that $0(G/S) \leq |S| \forall S \subseteq V$. Deduce that every 3-regular graph without cut edges has a perfect matching.

Or

(b) Define k-edge chromatic. Let G be a connected graph that is not an odd cycle. Then prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree atleast two.

14. (a) Define α', β'. If $\delta > 0$, prove that $\alpha' + \beta' = \nu$.

Or

(b) Define $r(k, \ell)$. Find the values of $r(1, \ell)$, $r(k, \ell)$, $r(2, \ell)$, $r(k, 2)$ and $r(3,3)$.

15. (a) Show that in a critical graph no vertex cut is a clique. Deduce that every critical graph is a block.

Or

(b) Show that if G is tree, then $\pi_k(G) = k(k-1)^{v-1}$.

SECTION C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let G be a graph with $\nu - 1$ edges. Show that the following are equivalent.

(i) G is connected

(ii) G is acyclic

(iii) G is a tree.

Or

(b) Define the terms k, k', δ. Show that in any graph $k \leq k' \leq \delta$. Give an example of a graph with $k = 1$, $k' = 3$, $\delta = 5$.

17. (a) Define the graph $C_{m,p}$. Let G be a simple graph with degree sequence $d_1 \leq d_2 \leq \ldots \leq d_p$. If G is non Hamiltonian, show that there exists an $m < \frac{p}{2}$ such that $d_m \leq m$ and $d_{p-m} < p - m$. Deduce that G is degree majorised by some $C_{m,p}$.

Or

(b) Define a Eulerian graph. Show that a nonempty connected graph G is Eulerian if and only if it has no vertices of odd degree. Also prove that a connected graph has an Euler trail if and only if it has atmost two vertices of odd degree.

18. (a) State and prove Vizing's theorem. If G is bipartite, show that $\psi' = \Delta$. Find a 4-edge proper colouring of $K_{3,4}$.

Or

(b) Define a matching in a graph and M-alternating path and M-augmenting path. Prove that a matching M is a maximum matching in G if and only if G contains no M-augmenting path.

19. (a) Show that for any two integers $k, \ell \geq 2$, $r(k,\ell) \leq r(k,\ell-1) + r(k-1,\ell)$. Show also that if $r(k,\ell-1)$ and $r(k-1,\ell)$ are both even then strict inequality holds.

Or

(b) Define the terms clique and m-partite graph. State and prove Turan's theorem.

20. (a) Define $\pi_k(G)$. Show that $\pi_k(G)$ is a polynomial in k of degree v with integer coefficients, leading term k^v and constant term zero. Show also that the coefficients of $\pi_k(G)$ alternate in sign. Find $\pi_k(G)$ where G is the cycle C_4.

Or

(b) Define a k-critical graph. State and prove Direc's theorem.

(7 pages) **Reg. No. :** ....

Code No. : 7854 **Sub. Code : PMAM 43**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2019.

Fourth Semester

Mathematics – Core

ADVANCED ALGEBRA — II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. What is the degree of $\sqrt{2} + \sqrt{3}$ over Q?

   (a) 1 (b) 2

   (c) 4 (d) None

2. If $a, b \in K$ are algebraic over F of degrees 8 and 3 respectively then the degree of $F(a, b)$ is

   (a) 5 (b) 11

   (c) 24 (d) 16

3. If $a \in K$ is a root of $p(x) \in F[x]$ of multiplicity $m > 1$, which of the following is not true?

(a) $(x-a)|p(x)$ (b) $(x-a)^m|p(x)$

(c) $(x-a)^{m+1}|p(x)$ (d) $(x-a)^{m+1} \nmid p(x)$

4. If F is of characteristic 3, then $f(x) = x^6 - x^3 + 1$ has

(a) distinct roots

(b) a multiple root

(c) no roots

(d) can't say anything about roots

5. Which of the following is true?

(a) $[K : F] = 0(G(K, F))$

(b) $O(G(K, F)) = n!$

(c) $G(K, F) = S_n$

(d) $[K : F] = n!$

6. If K is the field of complex numbers and F is the field of real numbers then $G(K, F)$

(a) does not exist

(b) is a group of order 2

(c) is K

(d) is F

7. If p is the characteristic of a finite field then the number of elements in F is

(a) 2

(b) m^p

(c) mp

(d) p^m

8. The multiplicative group of non-zero elements of a finite field

(a) is a permutation group

(b) is cyclic

(c) need not be cyclic

(d) is a subring

9. If H is the Hurwitz ring of integral quaternions and $a, a^{-1} \in H$, then $N(a) =$

(a) 0 (b) 1

(c) ∞ (d) not defined

10. If * denotes the adjoint operator then $(xy)^* =$

(a) $x^{-1}y^{-1}$ (b) $y^{-1}x^{-1}$

(c) $x^* y^*$ (d) $y^* x^*$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $T = \{\beta_0 + \beta_1 a + ... + \beta_{n-1} a^{n-1} | \beta_0, \beta_1 .... \beta_{n-1} \in F\}$, where $a \in K$ is algebraic of degree n, show that $T = F(a)$.

Or

(b) Show that the elements in K which are algebraic over F form a subfield of K.

12. (a) State and prove the Remainder Theorem.

Or

(b) If F is of characteristics $p \neq 0$ and if $f(x) \in F[x]$ is such that $f'(x) = 0$, prove that $f(x) = g(x^p)$ for some polynomial $g(x) \in F[x]$.

13. (a) If K is a finite extension of F, show that $G(K,F)$ is a finite group and $O(G(K,F)) \leq [K:F]$.

Or

(b) Given K is a normal extension of F and H is a subgroup of $G(K,F)$. If K_H is the fixed field of H, prove that (i) $[K:K_H] = O(H)$ and (ii) $H = G(K,K_H)$.

14. (a) If the field F has p^m elements, show that F is the splitting field of the polynomial $x^{p^m} - x$.

Or

(b) If $\alpha \neq 0$, $\beta \neq 0$ are two elements of a finite field F, show that we can find elements a and b in F such that $1 + \alpha a^2 + \beta b^2 = 0$.

15. (a) State and prove the Lagrange Identity.

Or

(b) Given C is the field of complex numbers and the division ring D is algebraic over C. Prove that $D = C$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If L is a finite extension of K and if K is a finite extension of F, show that L is a finite extension of F and $[L : F] = [L : K][K : F]$.

Or

(b) Prove that $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

17. (a) Prove that any two splitting fields of the same polynomial over a given field F are isomorphic by an isomorphism leaving every element of F fixed.

Or

(b) If F is of characteristic O and if a, b are algebraic over F, then show that there exists an element $C \in F(a,b)$ such that $F(a, b) = F(c)$.

18. (a) State and prove the fundamental theorem of Galois Theory.

Or

(b) Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.

19. (a) State and prove Wedderburn's Theorem on Finite Division Rings.

Or

(b) Prove that for every prime number p and every positive integer m there is a unique field having p^m elements.

20. (a) State and prove Left-Division Algorithm in the Hurwitz ring H of integral quaternions.

Or

(b) State and prove Lagrange's Four-Square theorem.

(8 pages) **Reg. No. :** ....

Code No. : 7845 **Sub. Code : PMAM 32**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Third Semester

Mathematics – Core

TOPOLOGY – I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The limit points of (0, 1) in R (indiscrete topology) are

(a) {0, 1} (b) [0, 1]

(c) (0, 1) (d) R

2. Let $X = \{a,b,c\}; \tau = \{X,\phi,\{a\},\{b\},\{a,b\}\}$. The sequence $(a_n); a_n = \begin{cases} a\, if\, n\, is\, odd \\ b\, if\, n\, is\, even \end{cases}$ coverages to

(a) a, b (b) a, b, c

(c) c only (d) none

3. The function f:R → R defined by f(x) = 7+4x is

(a) 1–1 but not onto

(b) onto but not 1–1

(c) bijection but not continuous

(d) homeomorphism

4. Let f:R → R be a continuous map. Then for every subset A of R

(a) $f(\overline{A}) = \overline{f(A)}$ (b) $f(\overline{A}) \not\subset \overline{f(A)}$

(c) $f(\overline{A}) \supseteq \overline{f(A)}$ (d) $f^{-1}(\overline{A})$ is closed

5. Which one of the following is metrizable

(a) [0, 1] as a subspace of ℝ with standard topology

(b) ℝ with K topology

(c) ℝ with lower limit topology

(d) none of these

6. Which one of the following is not metrizable

(a) {0,1,2} with indiscrete topology

(b) ℚ with co countable topology

(c) ℝ with standard topology

(d) none of these

7. Which one of the following is compact

(a) $X = (0,1] \cup (2,3)$ as a subspace of R with standard topology

(b) $X = (0,1]$ with discrete topology

(c) R with finite compliment topology

(d) Q with co countable topology

8. Which one of the following is connected

(a) $X = (0,1] \cup (2,3)$ with indiscrete topology

(b) $X = (0,1]$ with discrete topology

(c) $S = \{(x, y, z)\}$ with finite compliment topology

(d) Q with co countable topology

9. Which one of the following is not locally compact

(a) $X = [0,3] \cup [4,6]$ with indiscrete topology

(b) R with co countable topology

(c) $X = [0,3] \cup [4,6]$ with standard topology as a subspace of R

(d) Q with finite compliment topology

10. ———— is a limit point compact space

(a) $\mathbb{R}$ with standard topology

(b) $\mathbb{Z}_+ \times Y$ where $Y = \{a,b\}$ with indiscrete topology

(c) $\mathbb{R}$ with co countable topology

(d) $\mathbb{R}$ with lower limit topology

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that a subset A is closed in the subspace Y of a topological space X if and only if A is the intersection of a closed set of X with Y.

Or

(b) Let $\{\tau_\alpha\}$ be a collection of topologies on X. Show that $\bigcap_\alpha \tau_\alpha$ is a topology on X. What about $\bigcup_\alpha \tau_\alpha$?

12. (a) Prove that a space is Hausdorff if and only if the diagonal $D = \{(x,x) : x \in X\}$ is closed in the product space X×X.

Or

(b) Let $\{X^\alpha\}$ be a collection of topological spaces. Prove that if each X_α is Hausdorff, then $\prod X_\alpha$ is also hausdorff in both box and product topologies. Also prove that if $A_\alpha \subset X_\alpha$ for each α then $\prod \overline{A_\alpha} = \overline{\prod A_\alpha}$ in both box and product topologies.

13. (a) Prove that $\mathbb{R}^{\omega}$ is not metrizable in the box topology.

Or

(b) Define uniform topology on $\mathbb{R}^{J}$. Prove that the uniform topology on $\mathbb{R}^{J}$ is finer than the product topology and coarser than the box topology.

14. (a) Prove that $\mathbb{R}^{\omega}$ is not connected in the box topology but is connected in product topology.

Or

(b) Show that the compact subset of a Hausdorff space is closed. Give an example of a closed subset of a Hausdorff space which is not compact.

15. (a) Define limit point compact. Prove that compactness implies limit point compactness but not the converse.

Or

(b) Let X be a locally compact Hausdorff; let A be a subspace of X. If A is closed in X or open in X, then prove that A is locally compact.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) Define finite complement topology and a convergent sequence in a topological space. What are the closed sets in it? In the finite compliment topology on R, to which point or points does the sequence $x_{n = 1/n}$ converges.

Or

(b) Define K topology $\mathbb{R}_K$ and lower limit topology $\mathbb{R}_L$. Prove that they are not comparable but both are strictly finer than standard topology.

17. (a) Let $f:X \to Y$ be a function between topological spaces. Show that the following are equivalent.

(i) f is continuous

(ii) $f(\overline{A}) \subset \overline{f(A)}$ for every subset A of X.

(iii) for every closed set B of Y, $f^{-1}(B)$ is closed in X

(iv) For each $x \in X$ and each neighborhood V of f(x), there is a neighborhood U of x such that $f(U) \subset V$.

Or

(b) Define product topology and box topology. Let $f : A \to \prod_{\alpha \in J} X_\alpha$ be given by the equation $f(a) = (f_\alpha(a))\alpha \in J$ where $f_\alpha : A \to X_\alpha$ for each α. Let $\prod X_\alpha$ has the product topology. Prove that the function f is continuous if and only if each f_α is. Hence show that the product and box topologies are different on R^ω.

18. (a) (i) State and prove sequence lemma.

(ii) Define uniform convergence of a sequence. State and prove uniform limit theorem.

Or

(b) (i) Let $f : X \to Y$; let X and Y be metrizable with metrics d_x and d_y respectively. Then prove that continuity of f is equivalent to the requirement that given $x \in X$ and given $\in > 0$, there exists $\delta > 0$ such that $d_X(x, y) < \delta \Rightarrow dy(f(x), f(y)) < \in$.

(ii) Let $f : X \to Y$. Prove that if f is continuous, then '(x_n) converges to x $\Rightarrow f(x_n)$ converges to f(x)' and the converse holds if X is metrizable.

19. (a) (i) Prove that the product of finitely many compact spaces is compact.

(ii) Let Y be a subspace of X. The Y is compact if and only if every covering by sets open in X contains a finite sub collection covering Y.

Or

(b) Define a connected space. Prove that finite Cartesian product of connected spaces is connected. Also prove that union of connected subspaces of a topological space that have a common point is also connected.

20. (a) Define one point compactification. Define locally compact space. Prove that a space X is locally compact Hausdorff space if and only if it has a unique one point compactification.

Or

(b) If X ix a metrizable topological space then prove that the following are equivalent.

(i) X is compact

(ii) X is limit point compact

(iii) X is sequentially compact

(8 pages) **Reg. No. : ....**

Code No. : 7130 **Sub. Code : PMAE 31**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Third Semester

Mathematics

Elective — ALGEBRAIC NUMBER THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The equation $ax + by = c$ with $(a, b) = g$ has at least one positive solution if

   (a) $g \mid c$ and $gc < ab$ (b) $c \mid g$ and $gc < ab$

   (c) $c \mid g$ and $gc > ab$ (d) $g \mid c$ and $gc > ab$

2. The number of integral solutions of $12x + 501y = 1$ is

   (a) 3 (b) 0

   (c) 1 (d) 2

3. The number of positive solutions of $x^2 + y^2 = z^2$ which are in geometric progression is

(a) 0 (b) 1

(c) ∞ (d) 2

4. With usual notations the values of $N'(1), P'(1)$ and $Q'(1)$ are

(a) 2, 2, 2 (b) 2, 1, 2

(c) 1, 2, 1 (d) 2, 2, 1

5. The value of the infinite continued fraction $\langle 2,2,2,2,2,2,\ldots\rangle$ is

(a) $1-\sqrt{2}$ (b) $1+\sqrt{2}$

(c) $\sqrt{2}-1$ (d) $\sqrt{2}$

6. The infinite continued fraction of $\sqrt{3}$ is

(a) $\langle 1, 1, 1, 2, 1, 2, 1, 2,\ldots\rangle$

(b) $\langle 2, 1, 1, 2, 1, 2, 1, 2,\ldots\rangle$

(c) $\langle 0, 1, 1, 2, 1, 2, 1, 2, 1,\ldots\rangle$

(d) $\langle 1, 1, 2, 1, 2, 1, 2\ldots\rangle$

7. The units of the rational number field Q are

(a) $\pm i$ (b) ± 1

(c) $\pm\sqrt{2}$ (d) $\pm\sqrt{3}$

8. The nth convergent of $\frac{1}{x}$ is the ________ of the convergent of x if x is any real number > 1.

(a) approximation, $(n+1)$

(b) reciprocal, $(n+1)$ st

(c) approximation, $(n-1)$ st

(d) reciprocal, $(n-1)$ st

9. Which one of the following is not the correct answer? $Q\left(\frac{a+b\sqrt{m}}{c}\right) =$

(a) $Q\,(a+b\sqrt{m})$

(b) $Q\,(a-b\sqrt{m})$

(c) $Q\,(b\sqrt{m})$

(d) $Q\,(\sqrt{m})$

10. The value of $N\left(\frac{5+3+\sqrt{2}}{4}\right)$ in $Q\,(\sqrt{2})$ is ________.

(a) $\frac{7}{16}$ (b) $\frac{11}{16}$

(c) $\frac{7}{4}$ (d) $\frac{43}{16}$

SECTION B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that the equation $ax + by = c$ with a and b are integers has integral solution if $(a, b) \mid c$. If (x_1, y_1) is a particular solution of $ax + by = c$. Find the general solution of the same.

Or

(b) Solve : $x + 2y + 3z = 10$.

12. (a) Let n be an integer, $n > 1, n \equiv 1 \pmod 4$. Prove that if n is a prime then $4x^2 + y^2 = n$ has exactly one non negative solution and it is a primitive solution. If n is not a prime then $4x^2 + y^2 = n$ has either no primitive solutions, more than one non negative primitive solution or it has one non negative primitive solution and at least one non negative primitive solution.

Or

(b) Prove that if r and s are arbitrary integers of opposite parity with $r > s > 1$ and $(r, s) = 1$ then $x = r^2 - s^2, y = 2rs, z = r^2 + s^2$ is a positive primitive solution of $x^2 + y^2 = z^2$.

13. (a) With usual notations, prove that for any positive real number x,

$$\langle a_0, a_1, \ldots a_{n-1}, x \rangle = \frac{x h_{n-1} + h_{n-2}}{x k_{n-1} + k_{n-2}}.$$

Or

(b) Prove that any two infinite simple continued fractions converge to different values.

14. (a) If α is any algebraic number prove that there is a rational b such that $b\alpha$ is an algebraic number.

Or

(b) Prove that the reciprocal of a unit is a unit and the units of an algebraic number field form a multiplicative group.

15. (a) If α, β, γ are in $Q(\sqrt{m})$ then prove the following :

(i) $N(\alpha\, \beta) = N(\alpha)\, N(\beta)$

(ii) $N(\alpha) = 0$ if $\alpha = 0$.

(iii) if γ is an integer in $Q(\sqrt{m})$ then $N(\gamma) = \pm 1$ iff γ is a unit.

Or

(b) Prove that there are infinitely many units in any real quadratic field.

SECTION C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Discuss the procedure of solving the equation $a_1x_1 + a_2x_2 + \dots + a_kx_k = c, k > 2$.

Or

(b) (i) Find all positive solutions of $5x + 3y = 52$.

(ii) Prove that the equation $ax + by = a + c$ is solvable iff the equation $ax + by = c$ is solvable.

17. (a) Prove that the positive primitive solutions of $x^2 + y^2 = z^2$ with y even are $x = r^2 - s^2 y = 2rs, z = r^2 + s^2$ where r and s are arbitrary integers of opposite parity with $r > s > 0$ and $(r, s) = 1$.

Or

(b) Prove that the only integral solutions of $x^4 + y^4 = z^2$ are the solutions $x = 0$, $y, z = \pm x^2$ and $y = 0, z = \pm x^2$.

18. (a) Prove that if a, b, c do not have the same sign and that $-bc, -ac, -ab$ are quadratic residues modulo a, b, c respectively then the equation $ax^2 + by^2 + cz^2 = 0$ has a solution in integers x, y, z not all zero where a, b, c are non zero integers such that the product abc is square free.

Or

(b) Prove that the value of any infinite simple continued fraction $\langle a_0, a_1, ... a_{n-1}, x \rangle$ is irrational.

19. (a) If a/b is a rational number with positive denominator such that $|\xi - a/b| < |\xi - h_n/k_n|$ for some $n \geq 1$, prove that $b > k_n$. In fact $|\xi b - a| < |\xi k_n - h_n|$ for some $n \geq 0$, then $b \geq k_{n+1}$.

Or

(b) Prove the following :

(i) for $n \geq 0 \left|\xi - \frac{h_n}{k_n}\right| < \frac{1}{k_n k_{n+1}}$ and $|\xi k_n - h_n| < \frac{1}{k_{n+1}}$

(ii) The convergents $\frac{h_n}{k_n}$ are successively closure to ξ, that is $\left|\xi - \frac{h_n}{k_n}\right| < \left|\xi - \frac{h_n - 1}{k_n - 1}\right|$. In fact the stronger inequality $|\xi k_n - h_n| < |\xi k_{n-1} - h_{n-1}|$ holds.

20. (a) Prove that every quadratic field is of the form $Q(\sqrt{m})$ where m is a square free rational integer, positive or negative but not equal to 1. Numbers of the form $a+b\sqrt{m}$ with rational integers a and b are integers of $Q(\sqrt{m})$. These are the only integers of $Q(\sqrt{m})$ if $m \equiv 2$ or $3 \pmod 4$. If $m \equiv 1 \pmod 4$ the numbers $\left(\frac{a+b\sqrt{m}}{2}\right)$ with odd rational integers a and b are also integers of $Q(\sqrt{m})$ and there are no further integers.

Or

(b) Let m be a negative square free rational integer. Prove that the field $Q(\sqrt{m})$ has units ± 1 and these are the only units except in the case $m=-1$ and $m=-3$. The units for $Q(i)$ are ± 1 and $\pm i$. The units for $Q(\sqrt{-3})$ are ± 1, $\frac{1 \pm \sqrt{-3}}{2}$ and $\frac{-1 \pm \sqrt{-3}}{2}$.

(8 pages) **Reg. No. :** ....

Code No. : 7846 **Sub. Code : PMAM 33**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Third Semester

Mathematics — Core

ADVANCED ALGEBRA — I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If V is finite dimensional and W is a subspace of V, then $\dim A(W) =$ ————

(a) $\dim V - \dim W$

(b) $\dim W - \dim V$

(c) $\dim \hat{V} - \dim \hat{W}$

(d) None

2. If $u, v, \in V$ then u is said to be orthogonal to v if ————

(a) $(u,v) = 0$ (b) $(u,v) \neq 0$

(c) $(u,v) = 1$ (d) $(u, v) \neq 1$

3. If $T \in A(V)$ and if $S \in A(V)$ is regular then ————

(a) $r(T) < r(STS^{-1})$

(b) $r(T) > r(STS^{-1})$

(c) $r(T) = r(STS^{-1})$

(d) $r(STS^{-1}) > r(S)$

4. If V is finite dimensional over F, and if $T \in A(V)$ is singular then there exists an $S \neq 0$ in $A(V)$ such that ————

(a) $ST = TS = I$ (b) $ST = TS = 0$

(c) $ST \neq TS$ (d) $ST = TS = \{0\}$

5. A subspace W of V is invariant under $T \in A(V)$ if ————

(a) $WT \supset W$ (b) $WT = W$

(c) $WT \subset W$ (d) $WT \neq W$

6. If $T \in A(V)$ is nilpotent, then ———— is called the index of nipotence of T if $T^k = 0$ but $T^{k-1} \neq 0$

(a) $K-1$ (b) K

(c) $K+1$ (d) $K-2$

7. If A is invertible then $tr(A \subset A^{-1}) =$

(a) $tr\,A$ (b) $tr\,A^{-1}$

(c) $tr\,AA^{-1}$ (d) $tr\,C$

8. For $A, B \in F_n$, $\det(AB) =$ ————

(a) $\det A + \det B$ (b) $\det A - \det B$

(c) $(\det A)(\det B)$ (d) none

9. If $T \in A(V)$ then the Hermitian adjoint of T, T^* is defined by $(uT, v) =$ ———— for all $u, v \in V$

(a) (uT^*, v) (b) (u, vT^*)

(c) (u, Tv) (d) (v, T_u^*)

10. $T \in A(V)$ is unitary if and only if ————

(a) $TT^* = 0$ (b) $TT^* \neq 0$

(c) $TT^* = T^*T$ (d) $TT^* = 1$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If V is finite dimensional, then prove that ψ is an isomorphism of V onto $\hat{V}$.

Or

(b) If $u, v \in V$, then prove that $|(u, v)| \leq \|u\|\|v\|$.

12. (a) Show that if V is finite dimensional over F, then $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not zero.

Or

(b) Prove that the element $\lambda \in F$ is a characteristic root of $T \in A(V)$ if and only if for some $v \neq 0$ in V, $vT = \lambda v$.

13. (a) If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F then show that T satisfies a polynomial of degree n over F.

Or

(b) Let $T \in A(V)$ have all its distinct characteristic roots $\lambda_1, \lambda_2, \ldots, \lambda_k$ in F, then a basis of V can be found in which the matrix T is of the form $\begin{pmatrix} J_1 & & & \\ & J_2 & & \\ & & \ddots & \\ & & & J_k \end{pmatrix}$ where each $J_i = \begin{pmatrix} Bi_1 & & & \\ & Bi_2 & & \\ & & \ddots & \\ & & & Bi_{r_i} \end{pmatrix}$ and where $Bi_1, Bi_2, \ldots Bi_{r_i}$ are basic Jordan blocks belonging to λ_i.

14. (a) For $A, B \in F_n$ and $\lambda \in F$, prove the following :

(i) $tr(\lambda A) = \lambda tr A$

(ii) $tr(A + B) = trA + trB$

(iii) $tr(AB) = tr(BA)$.

Or

(b) Prove that every $A \in F_n$ satisfies its secular equation.

15. (a) Prove that if $\{v_1, v_2, \ldots, v_n\}$ is an orthonormal basis of V and if the matrix of $T \in A(V)$ in this basis is (α_{ij}) then the matrix of T^* in this basis is (β_{ij}) where $\beta_{ij} = \overline{\alpha_{ji}}$.

Or

(b) Show that if N is normal and $AN = NA$ then $AN^* = N^* A$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Let V and W be two vector spaces over F of dimensions m and n respectively. Prove that $Hom(V, W)$ is a vector space over F. Also find the dimension of $Hom(V, W)$ over F.

Or

(b) Let V be a finite dimensional inner product space then prove that V has an orthogonal set as a basis.

17. (a) Prove that if V is finite dimensional over F, then $T \in A(V)$ is regular iff T maps V onto V.

Or

(b) Show that if V is n-dimensional over F and if $T \in A(V)$ has the matrix $m_1(T)$ in the basis $v_1, v_2, \ldots v_n$ and the matrix $m_2(T)$ in the basis $w_1, w_2, \ldots w_n$ of V over F, then there is an element $C \in F_n$ such that $m_2(T) = Cm_1(T)C^{-1}$. Infact, if S is the linear transformation of V defined by $v_i s = w_i$ for $i = 1, 2, \ldots n$ then C can be chosen to be $m_1(s)$.

18. (a) If $T \in A(V)$ has all its characteristics roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

Or

(b) Show that two nilpotent linear transformations are similar if and only if they have the same invariants.

19 (a) State and prove Cramer's rule.

Or

(b) Show that A is invertible if and only if $\det A \neq 0$.

20. (a) If $T \in A(V)$ then prove that $T^* \in A(V)$. More over

(i) $(T^*)^* = T$

(ii) $(S+T)^* = S^* + T^*$

(iii) $(\lambda S)^* = \bar{\lambda} S^*$

(iv) $(ST)^* = T^* S^*$.

Or

(b) Prove that if N is a normal linear transformation on V, then there exists an orthonormal basis consisting of characteristic vectors of N, in which the matrix of N is diagonal, Equivalently, if N is a normal matrix there exists a unitary matrix U such that $UNU^{-1} (= UNU^*)$ is diagonal.

(8 pages) **Reg. No. : ....**

Code No. : 7126 **Sub. Code : PMAM 32**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Third Semester

Mathematics — Core

TOPOLOGY — I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. Which one of the following is not a topology on $X = \{a, b, c, d\}$?

   (a) $\{\phi, X, \{a\}, \{c\}, \{a, c\}\}$

   (b) $\{\phi, X\}$

   (c) $\{\phi, X, \{a, c\}, \{c, d\}\}$

   (d) $\{\phi, X, \{a\}, \{a, b\}, \{a, b, c\}\}$

2. If $C = \{0\} \cup (1, 2)$ in R then C' is

(a) $\{0\}$ (b) $[1, 2]$

(c) $[0, 2]$ (d) C

3. If $f : R \to R$ is given by $f(x) = 3x + 1$ then $g = f^{-1}$ is given by

(a) $g(y) = \frac{y+1}{3}$ (b) $g(y) = y - 3$

(c) $g(y) = \frac{y-1}{3}$ (d) $g(y) = 3y - 1$

4. Which one of the following is not continuous?

(a) $f : R \to R$ defined by $f(x) = x \ \forall x \in R$

(b) $f : R \to R_l$ defined by $f(x) = x \ \forall x \in R$

(c) $f : R_l \to R$ defined by $f(x) = x \ \forall x \in R$

(d) $f : R_l \to R_l$ defined by $f(x) = x \ \forall x \in R$

5. The basis element (3, 7) for the order topology in R is the following basis element for the metric topology

(a) $B(3, 7)$ (b) $B(5, 4)$

(c) $B(5, 2)$ (d) $B(3, 4)$

6. The square metric ρ on R^n is defined by

(a) $\rho(x,y)=[(x_1-y_1)^2+.....+(x_n-y_n)^2]^{1/2}$

(b) $\rho(x,y)=\max\left\{|x_1,-y_1|,....,|x_n-y_n|\right\}$

(c) $\rho(x,y)=\min\left\{|x_1,-y_1|,....,|x_n-y_n|\right\}$

(d) $\rho(x,y)=\sup\left\{\overline{d}(x_i,y_i)/i\right\}$

7. Let $X=\{a,b,c\}$. Which one of the following topology is connected

(a) $\{\phi, X,\{a\},\{b,c\}\}$

(b) $\{\phi, X,\{b\},\{c\},\{a\},\{a,b\},\{a,c\},\{b,c\}\}$

(c) $\{\phi, X,\{b\},\{a,c\}\}$

(d) $\{\phi, X,\{a\},\{b\},\{a,b\}\}$

8. Which one of the following is an open cover for R?

(a) $\{(n, n+1), n\in z\}$

(b) $\{(n, n+3)/n\in z\}$

(c) $\{\{n\}/n\in z\}$

(d) $\{[n, n+2]/n\in z\}$

9. Which one of the following is not locally compact?

(a) Real line R

(b) The subspace Q of rational number

(c) The space R^n

(d) Any simply ordered set having the least upper bound property.

10. A space X is homeomorphic to an open subspace of a compact Hausdorff space if and only if X is

(a) Compact Hausdorff

(b) Locally compact Hausdorff

(c) Compact and connected

(d) Countably compact

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $\{\tau_\alpha\}$ is a family of topologies on X, show that $\cap \tau_\alpha$ is a topology on X. Is $\cup \tau_\alpha$ a topology on X? Justify your answer.

Or

(b) Define a Hausdorff space and show that every finite point set in a Hausdorff space X is closed.

12. (a) State and prove the pasting lemma.

Or

(b) Using basis elements, compare the box and product topologies.

13. (a) Let X be a metric space with metric d. Define $\overline{d}: X \times X \to R$ by the equation $\overline{d} < (x, y) = \min\{d(x, y), 1\}$. Prove that $\overline{d}$ is a metric that induces the same topology as d.

Or

(b) State and prove the sequence lemma.

14. (a) Prove that the union of a collection of connected subspaces of X that have a point in common is connected.

Or

(b) Prove that the image of a compact space under a continuous map is compact.

15. (a) Prove that compactness implies limit point compactness.

Or

(b) Let X be locally compact Hausdorff ; Let A be a subspace of X. If A is closed in X or open in X, prove that A is locally compact.

PART C — $(5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Define the standard topology, lower limit topology and K-topology on R and find the relation between these topologies.

Or

(b) Define the closure of a set and the limit points of a set A and show that $\overline{A} = A \cup A'$.

17. (a) Let X and Y be topologies spaces; let $f: X \to Y$. Prove that following are equivalent

(i) f is continuous

(ii) For every subset A of X, one has $f(\overline{A}) \subseteq \overline{f(A)}$

(iii) For every closed set B of Y, the set $f^{-1}(B)$ is closed in X

(iv) For each $x \in X$ and each neighbourhood V of $f(x)$, there is a neighbourhood U of x such that $f(U) \subseteq V$.

Or

(b) Let X and X' denote a single set in the two topologies τ and τ' respectively. Let $i : X' \to X$ be the identity function.

(i) Show that i continuous $\Leftrightarrow \tau'$ is fines then τ.

(ii) Show that i is a homeomorphism $\Leftrightarrow$ $\tau' = \tau$.

18. (a) Prove that the topologies on R^n induced by the euclidean metric d and the square metric ρ are the same as the product topology on R^n.

Or

(b) State and prove uniform limit theorem.

19. (a) Prove that a finite Cartesian product of connected spaces is connected.

Or

(b) Prove that every compact subspace of a Hausdorff space is closed.

20. (a) Show that the sequentially compact implies compactness.

Or

(b) Define the one point compactification of a topological space X. Let X be a Hausdorff space. Prove that X is locally., compact if and only if given x in X and given a neighborhood U of x, there is a neighbourhood V of x such that $\overline{V}$ is compact and $\overline{V} \subseteq U$.

(6 pages) **Reg. No. :** ....

Code No. : 7855 **Sub. Code : PMAM 44**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Fourth Semester

Mathematics — Core

TOPOLOGY — II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. A subset A of a space X is said to be dense in X if ————.

   (a) $\overline{A} = A$ (b) $\overline{A} = X$

   (c) $\overline{A} = \varphi$ (d) None

2. If X is Lindelof and Y is compact then ———— is not Lindelof.

   (a) $X \times Y$ (b) $X \cup Y$

   (c) $X \cap Y$ (d) None of these

3. Every well-ordered set X is ———— in the order topology.

   (a) dense (b) compact

   (c) regular (d) normal

4. The product $s_\Omega \times \overline{s}_\Omega$ is ————.

   (a) Normal (b) Hausdorff

   (c) Not normal (d) Uncountable

5. If X is ———— if and only if X has a countable bias.

   (a) Compact (b) Hausdorff

   (c) Metrizable (d) All

6. A subspace of a completely regular space is ————.

   (a) Completely regular

   (b) Regular

   (c) Normal

   (d) Separable

7. The collection $\mathbb{B} = \{ (0,\ 1/n) \cap \in z_f \}$ is locally finite in ————.

   (a) $\mathbb{R}$ (b) $\left(0, \frac{1}{n}\right)$

   (c) $(0, 1)$ (d) None

8. The collection $A = \{(n, n+2) / n \in z\}$ is locally fine in ————.

(a) (0, 2)
(b) (0, n)
(c) $\mathbb{R}$
(d) $\not\subset$

9. ———— are a Baire space.

(a) Irrational
(b) Rational
(c) Integers
(d) None of the above

10. If B has empty interior in X iff ———— is dense in X.

(a) $\overline{X} - B$
(b) $X - B$
(c) φ
(d) None

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that R^w with uniform topology is first countable but not second countable.

Or

(b) Let X be a topological space in which one-points sets are closed. Prove that

(i) X is regular iff given $x \in X$ and a neighbourhood U of x, there exist a neighbourhood V of $x \ni: \overline{V} \subseteq U$

(ii) X is normal iff given a closed set A of X and a open set U containing A, there exist an open set V containing $A \ni: \overline{V} \subseteq U$.

12. (a) Prove that every metrizable space is normal.

Or

(b) Prove that subspace of a completely regular space is completely regular.

13. (a) State and prove imbedding theorem.

Or

(b) Let X be metrizable. Show that the following are equivalent.

(i) X is bounded under every metric that gives the topology of X

(ii) Every continuous function $\varphi : X \to \mathbb{R}$ is bounded

(iii) X is limit point compact.

14. (a) Let X be a set, let D be a collection of subsets of X that is maximal with respect to the finite intersection property. Then prove that

(i) Any finite intersection of elements of D is an element of D

(ii) If A is a subset of X that intersects every element of D then A is an element of D.

Or

(b) (i) Define locally finite and given an example.

(ii) Define countably locally finite.

15. (a) If X is a Baire space iff given any countable collection $\{U_n\}$ of open sets in X, each of which is dense in X, prove that their intersection $\cap U_n$ is also dense in X.

Or

(b) Prove that any open subspace Y of a Baire space X is itself a Bair space.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Show that the space $\mathbb{R}_\ell$ satisfies all the countability except the second.

Or

(b) Show that the sorgen frey plane $\mathbb{R}_\ell^2$ is not normal.

17. (a) State and prove Urysohn lemma.

Or

(b) Show that a connected regular space having more than one point is uncountable.

18. (a) State and prove Urysohn metrization theorem.

Or

(b) State and prove Tietze extension theorem.

19. (a) State and prove Tychonoff theorem.

Or

(b) Let X be a metrizable space. If A is an open covering of X, then prove that there is an open covering δ of X refining A that is countably locally finite.

20. (a) State and prove Baire category theorem.

Or

(b) Let $c_1 \supset c_2 \supset \ldots$ be a nested sequence of nonempty closed sets in the complete metric space. If diam $c_n \to 0$, then show that $\cap c_n \neq \phi$.

———————

(8 pages) **Reg. No. :** ....

Code No. : 7128 **Sub. Code : PMAM 34**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Third Semester

Mathematics — Core

OPERATIONS RESEARCH

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — $(10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. Which of the following method gives solution closer to the optimum is

   (a) N-W method

   (b) Vogel's method

   (c) Least-cost method

   (d) None

2. When the cost matrix of an assignment problem is nA a square matrix, the assignment problem is called as

(a) Balanced A.P. (b) Unbalanced A.P.

(c) Degenerate A.P. (d) None

3. Floyd's algorithm is used so find the shortest route between ———— in the net work

(a) Source node and every other node

(b) Any two nodes

(c) Starting node and end node

(d) None

4. Free float of an activity is $FFi\,j =$ ————

(a) $Esj - ESi - Dij$ (b) $\dot{E}Sj + ESi - Dij$

(c) $ESj - ESi + Dij$ (d) None

5. Which method is more successful than the other methods in solving ILP

(a) Cutting plane method

(b) B and B mehod

(c) Additive algorithm

(d) None

6. The additive algorithm was developed in the ——————— year

(a) 1965 (b) 1975

(c) 1985 (d) 1955

7. The interest for money held up in the inventory is included is

(a) Setup cost (b) Holding cost

(c) Shortage cost (d) Ordering cost

8. In constant rate demand with instantaneous replenishment and no shortage model $>^{*}_{-}$ –

(a) $\frac{DK}{y}+\frac{yk}{2}$ (b) $\sqrt{\frac{2DK}{h}}$

(c) $\frac{yh}{2}$ (d) $\sqrt{2DKh}$

9. An arrival chooses not to join the queue even if there is space to join. The phenomenon is called

(a) Balking (b) Reneging

(c) Jock Gering (d) Queuring

10. In $(M/M/C):(GD/N/\infty), C \leq N, \lambda eff$

(a) $(1-P_1)\lambda$ (b) $(1-P_N)\lambda$

(c) $(1-P_1)\mu$ (d) $(1-P_n)\mu$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Give the simplex method explanation of the method of multipliers.

Or

(b) Describe the Hungarian algorithm.

12. (a) Write the floyd's and algorithm.

Or

(b) Discuss the critical path computations.

13. (a) Write the cutting plane algorithm.

Or

(b) Convert the following 0-1 problem satisfying the starting requirements of the additive algorithms

Maximize $Z=3x_1-5x_2$

Subject to

$$x_1+x_2=5$$
$$4x_1+6x_2\geq 4$$
$$x_1, x_2=(0, 1)$$

14. (a) Neon lights on the U of A campus are replaced at the rate of 100 units perday. The physical plant orders of the neon lights periodically. If cost \$. 100 to initiate a purchase order. A neon light kept in storage is estimated to cost about \$. 0.2 per day. The lead time between placing and receiving an order is 12 days. Determine the optimal inventory policy for ordering the neon lights.

Or

(b) The daily demand for an item during a single period occurs instantaneously at the rate start of the period. The pdf of the demand in uniform between 0 and 10 units. The unit holding cost of the item during the period is \$ 50 and the unit penalty cost for running out of stock is \$ 4.50. The unit purchase cost is \$ 50. A fixed cost of \$ 25 is incurred each time an order is placed. Determine the optimal inventory policy for the item.

15. (a) Describe pure death model.

Or

(b) Derive the measure of performance Ls, Lq, Ws and Wq in (M/M/1) : (GD/N/∞).

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions choosing either (a) or (b).

16. (a) In the transportation problem in the following table the total demand exceeds the total supply. Suppose that the penality cost per unit of the unsatisfied demand are \$5, \$3, \$2 for destination. 1, 2 and 3 respectively. determine the optimum solution

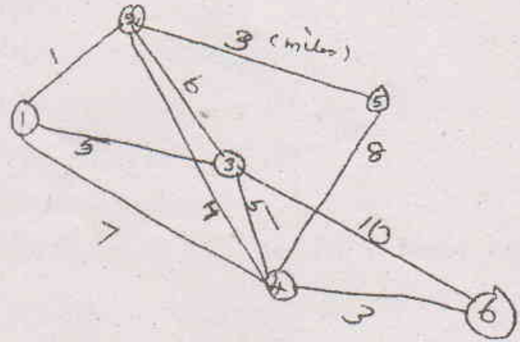
\$5	\$1	\$7	10
\$6	\$4	\$6	80
\$3	\$2	\$5	15
75	20	50	

Or

(b) Solve the assignment problem

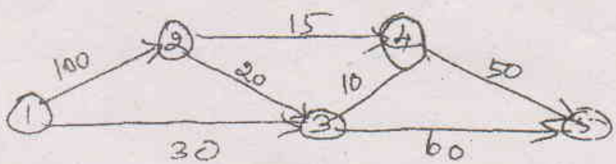
\$1	\$4	\$6	\$3
\$9	\$7	\$10	\$9
\$4	\$5	\$11	\$7
\$8	\$7	\$8	\$5

17. (a) Determine the minimal spanning tree of the network given below



Or

(b) Find the shortest route from city 1 to each of the remaining four cities.



18. (a) Solve the following ILP using B and B algorithms.

Maximize $Z = 5x_1 + 4x_2$

Subject to

$x_1 + x_2 \leq 5$

$10x_2 + 6x_2 \leq 45$

$x_1, x_2 \geq 0$ and integer.

Or

(b) Solve the following using additive algorithm

Maximize $Z = 3x_1 + x_2 + 3x_3$

Subject to

$$-x_1 + 2x_2 + x_3 \le 4$$
$$4x_2 - 3x_3 \le 2$$
$$x_1 - 3x_2 + 2x_3 \le 3$$
$$x_1, x_2, x_3 = (0,1)$$

19. (a) Explain multi-item EOQ with storage limitation.

Or

(b) In a single period inventory situation, the unit purchasing cost of a product is \$ 10, and its holding cost is \$ 10, and its holding cost is \$ 1. If the order quantity is 4 units find the permissible range of the penality cost implied by the optimal conditions. Assume that the demand occurs instantaneously at the start of the period and that demand pdf is given as

D	0.	1	2	3	4	5	6	7	8
f (D)	.05	.1	.1	.2	.25	.15	.05	.05	.05

20. (a) Explain $(M/M/1):(GD/N/\infty)$ model.

Or

(b) Explain $(M/M/e):(GD/\infty/\infty)$ model.

(7 pages) **Reg. No. : ....**

Code No. : 7129 **Sub. Code : PMAM 35**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Third Semester

Mathematics — Core

RESEARCH METHODOLOGY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. In the vast majority of cases that research will form a ——————.

   (a) Dissertation

   (b) Thesis

   (c) Either (a) or (b)

   (d) Degree

2. In the case of ——————— research, there tends to be more flexibility add less formality over dissertation proposals.

(a) Under Graduate (b) Post Graduate

(c) Doctoral Graduate (d) None

3. The word limits can vary for ———————.

(a) Theses (b) Dissertations

(c) Both (a) and (b) (d) But not (b)

4. The title of report needs to indicate ——————— of the research.

(a) nature (b) purpose

(c) Both (a) and (b) (d) But not (b)

5. If X has the moment – generating function $M(t) = e^{3t+32t^2}$, then X has a normal distribution with $\sigma^2 =$ ———————.

(a) 2 (b) 4

(c) 32 (d) 64

6. The mean of a chi-square distribution is $\mu =$ ———————.

(a) r (b) r^2

(c) $2r$ (d) $2r^2$

7. Let $X_1, X_2, ..., X_n$ denote a random sample of size n from a given distribution. Then S^2 = ——————.

(a) $\frac{1}{n}\sum_{1}^{n} X_i^2 - \overline{X}^2$ (b) $\frac{1}{n}\left(\sum_{1}^{n} X_i^2 - \overline{X}^2\right)$

(c) $\frac{1}{n}\sum_{1}^{n}\left(X_i^2 - \overline{X}\right)$ (d) $\left[\frac{1}{n}\sum\left(X_i - \overline{X}\right)\right]^2$

8. Let F have an F – distribution with parameters r_1 and r_2. Then $\frac{1}{F}$ has an F distribution with parameters ——————.

(a) r_1 and r_2 (b) r_2 and r_1

(c) $\frac{1}{r_1}$ and $\frac{1}{r_2}$ (d) $\frac{1}{r_2}$ and $\frac{1}{r_1}$

9. The sum of n mutually stochastically independent normally distributed variables has a —————— distribution.

(a) normal

(b) poisson

(c) exponential

(d) binomial

10. If $X_1, X_2, \ldots X_n$ denote a random sample of size $n \geq 2$ from a distribution that is $n(\mu, \sigma^2)$, then $\overline{X}$ is

(a) $n\left(\frac{\mu}{n}, \frac{\sigma^2}{n^2}\right)$

(b) $n\left(\mu, \frac{\sigma^2}{n}\right)$

(c) $n\left(\frac{\mu}{n}, \frac{\sigma^2}{n}\right)$

(d) $n(\mu, \sigma^2)$

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Write the difference between a dissertation and a thesis.

Or

(b) What is originality?

12. (a) Explain the way of writing acknowledgements.

Or

(b) Discuss the importance of literature review.

13. (a) If the random variable X is $n(\mu. \sigma^2), \sigma^2 > 0$, then prove that the random variable $W = \frac{X - \mu}{\sigma}$ is $n(0,1)$.

Or

(b) Let X be $\chi^2(10)$. Then find

(i) $\Pr(3.25 \leq X \leq 20.5)$

(ii) the value of a for which $\Pr(a < X) = 0.05$.

14. (a) Let X have the binomial p.d.f $f(x) = \frac{3!}{x!(3-x)!}\left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{3-x}$, $x = 0, 1, 2, 3$. Find the p.d.f of the random variable $Y = X^2$.

Or

(b) Let X be a random variable having p.d.f $\begin{aligned} f(x) &= 2x, 0 < x < 1 \\ &= 0 \quad \text{elsewhere} \end{aligned}$. Find the p.d.f of $Y = 8X^3$.

15. (a) Let $\overline{X}$ be the mean of a random sample of size 25 from a distribution that is $n(75, 100)$. Find $\Pr(71 < \overline{X} < 79)$.

Or

(b) Given that W is $n(0,1)$, the V is $\chi^2(r)$ with $r \geq 2$ and let W and V be stochastically independent. Find the variance of the variable $T = @\sqrt{r/V}$.

SECTION C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) What are the basic requirements of a thesis?

Or

(b) Write about the ethical considerations while writing a thesis.

17. (a) How do we conclude our thesis?

Or

(b) List the things to be included in the introduction part.

18. (a) (i) Let X be $n(2, 25)$. Find $\Pr(0 < X < 10)$ and $\Pr(-8 < X < 1)$.

(ii) Let X be $n(\mu, \sigma^2)$. Find $\Pr(\mu \cdot 2\sigma < X < \mu + 2\sigma)$.

Or

(b) Find the moment generating function of a normal distribution and hence find its mean and variance.

19. (a) Let X_1 and X_2 be two stochastically independent random variables that have poisson distributions with mean μ_1 and μ_2 respectively. Find the p.d.f of the random variable $Y = X_1 + X_2$.

Or

(b) Find the p.d.f of $Y_1 = \frac{1}{2}(X_1 - X_2)$ where X_1 and X_2 are stochastically independent random variables each being $\chi^2(2)$.

20. (a) Let $X_1, X_2, \ldots X_n$ denote a random sample of size $n \geq 2$ from a distribution that in $n(\mu, \sigma^2)$. Let $\overline{X}$ and S^2 be the mean and variance of this random sample respectively. Prove that $\frac{nS^2}{\sigma^2}$ is $\chi^2(n-1)$.

Or

(b) State and prove the central limit theorem.

(8 pages)

Reg. No. : ....

Code No. : 7133 **Sub. Code : PMAM 41**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Fourth Semester

Mathematics — Core

FUNCTIONAL ANALYSIS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. T is a bounded linear transformation if for every x and $k \geq 0$ s.t

   (a) $\|T(x)\| \leq k$ (b) $\|T(x)\| \leq kx$

   (c) $\|T(x)\| \leq k\|x\|$ (d) $\|T(x)\| < \infty$

2. $\|x\| - \|y\| - \|x - y\|$

   (a) $\leq$ (b) $\geq$

   (c) $<$ (d) $>$

3. For every $\mathfrak{C}$ in N^{x}, $F_{dx}(f) =$ ________.

(a) $F_x(\alpha f)$ (b) $(\alpha F_x)(f)$

(c) $F_x(F(\alpha))$ (d) $(xF_\alpha)(f)$

4. If X is a compact Hausdorff space, than $\mathfrak{C}(X)$ is reflexive if and only if

(a) X is an infinite set

(b) X is a finite set

(c) X is a bounded set

(d) X is not empty

5. If S is a non-empty subset of a Hilbert space than

(a) $S^{\perp\perp} = S^{\perp}$ (b) $S^{\perp\perp} = S^{\perp\perp\perp}$

(c) $S^{\perp} = S^{\perp\perp\perp}$ (d) $S^{\perp\perp\perp\perp} = S^{\perp\perp}$

6. If $\{e_i\}$ is on orthonormal set in a Hilbert space H then $\sum |(x, ei)|^2 \leq \|x\|^2$ for every $x \in H$ is called

(a) Schwarz inequality

(b) Bassel's inequality

(c) Triangle inequality

(d) Spectral inequality

7. Let $\{e_i\}$ be an orthonormal set in a Hilbert space H. Then $\{e_i\}$ is complete is equivalent to

(a) $x \perp \{e_i\} \Rightarrow x \neq 0$

(b) If x is an arbitrary vector is H then $x = \Sigma(x, e_i)\, e_i$

(c) Both (a) and (b) are equivalent

(d) Neither (a) nor (b) is true

8. Let H be a Hilbert space and T^* be adjoint of the operator T which one of the following is true

(a) $(\alpha T)^* = \alpha T^*$ (b) $(\alpha T)^* = \overline{\alpha} T^*$

(c) $(T_1 T_2)^* = T_1{}^* T_2{}^*$ (d) $\|T^* T\| = \|T\|$

9. If N is a normal operator on H then $\|N^2\| =$

(a) 1 (b) 0

(c) $\|N\|$ (d) $\|N\|^2$

10. If P is a projection on a Hilbert space H. Then one of the following is false

(a) P is a positive operator on H

(b) $\|Px\| \leq \|x\|$ for every $x \in H$

(c) $\|P\| \leq 1$

(d) None of them is true

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If M is a closed linear subspace of a normed linear space N and x_0 is a vector not in M, then prove that there exists a functional f_0 in N^* such that $f_0(m) = 0$ and $f_0(x) \neq 0$.

Or

(b) Let T be a linear transformation of a normal linear space N into N^*. Prove that T is continuous if and only if it is bounded.

12. (a) If P is a projection on a Banach space B and if M and N are its range and null space, then show that M and N are closed linear subspaces of B such that $B = M \oplus N$.

Or

(b) State and prove closed graph theorem.

13. (a) State and prove the uniform boundedness theorem.

Or

(b) Show that a closed convex set C of a Hilbert space H contains a unique vector of smallest norm.

14. (a) Show that $O^* = O$ and $I^* = I$. Use the later to show that if T is non-singular, then T^* is also non-singular and that in this case $(T^*)^{-1} = (T^{-1})^*$.

Or

(b) Prove that the adjoint operator $T \to T^*$ on $\mathcal{B}(H)$ has the following properties

(i) $(T_1 + T_2)^* = T_1^* + T_2^*$

(ii) $\|T^*T\| = \|T\|^2$.

15. (a) If T is an operator on H then show that T is normal if and only if its real and imaginary parts commute.

Or

(b) If T is normal, then prove that x is on eigen vector of T with eigen value λ if and only if x is an eigen vector of T^* with eigen value $\bar{\lambda}$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let M be a closed linear subspace of a normed linear space N. Prove that N/M is a normed linear space. Also prove that if N is a Banach space then so is N/M.

Or

(b) State and prove Hahn-Banach theorem.

17. (a) State and prove open mapping theorem.

Or

(b) If N is a normed linear space, then show that the closed unit sphere S^* and N^* is a compact Hausdorff space in the weak * Eopology.

18. (a) If B is a complex Banach space whose norms obeys the parallelogram law and if an inner product is defined by $4 < x, y >= \|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2$ then prove that B is a Hilbert space.

Or

(b) Let M and N be closed linear subspaces of a Hilbert space H. If $M \perp N$ then show that the linear sub space $M + N$ is closed and also prove that $H = M \oplus M^{\perp}$.

19. (a) Let H be a Hilbert space and let f be an arbitrary functional in H^*. Then show that there exists a unique vector y in H such that $f(x) = < x, y >$ for every vector x in H.

Or

(b) Prove that the self-adjoint operators in $\mathcal{B}$ (H) from a closed real linear subspace of $\mathcal{B}$ (H) and therefore a real Banach space which contains the identify transformation.

20. (a) Let T be an operator on H and prove the following

(i) T is singular $(z)\ 0 \in \sigma(T)$

(ii) If T is non-singular, then $\lambda \in \sigma(T)$ if and only if $\lambda^{-1} \in \sigma(T^{-1})$

(iii) If A is non-singular then $\sigma(ATA^{-1}) = \sigma(T)$

(iv) If $T^K = 0$ for some positive integer K, then $\sigma(T) = \{0\}$.

Or

(b) (i) If N_1 and N_2 are normal operators on H with the property that either commutes with the adjoint of the other, then show that $N_1 + N_2$ and $N_1 N_2$ are normal.

(ii) An operator T on H is normal if and only if $\|T^* x\| = \|Tx\|$ for every x.

(7 pages) **Reg. No. : ....**

Code No. : 6828 **Sub. Code : KMAM 42**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Fourth Semester

Mathematics

COMPLEX ANALYSIS

(For those who joined in July 2016 only)

Time : Three hours Maximum : 75 marks

SECTION A — $(10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. An analytic function $f(z)$ is independent of

   (a) z

   (b) $\overline{z}$

   (c) f

   (d) z^2

2. The Hadamard's formula is

(a) $R = \lim_{n \to \infty} \sup \sqrt[n]{|a_n|}$

(b) $\frac{1}{R} = \lim_{n \to \infty} \sup \sqrt[n]{|a_n|}$

(c) $R = \lim_{n \to \infty} \text{Inf} \sqrt[n]{|a_n|}$

(d) $\frac{1}{R} = \lim_{n \to \infty} \text{Inf} \sqrt[n]{a_n}$

3. (z_1, z_2, z_3, z_4) is the image of z_1 under the linear transformation which carries z_2, z_3, z_4 into

(a) $\infty, 0, 1$

(b) $0, 1, \infty$

(c) $1, 0, \infty$

(d) $1, 1, 1$

4. $\int_r f\,dz + \int_r f\,\overline{d}z$ is

(a) $2\int_r f\,dz$

(b) $2\int_r f\,dx$

(c) $\int_r f\,dx$

(d) $2\int_r f\,dy$

5. If C is a circle about a, then $\frac{1}{2\pi i}\int_c \frac{dz}{z-a}$ is

(a) 0 (b) 1

(c) 2 (d) ∞

6. $\frac{1}{2\pi i}\int_c \frac{f(\xi)}{(\xi - z)^2}\, d\xi$ is

(a) $f(0)$ (b) $f(z)$

(c) $f'(z)$ (d) $f'(0)$

7. For $\frac{e^z}{z}$, $z = 0$ is a

(a) simple pole (b) double pole

(c) zero of order 1 (d) zero of order 2

8. Residue of $\frac{z+1}{z(z-2)}$ at $z = 2$ is

(a) 0 (b) $\frac{2}{3}$

(c) $\frac{3}{2}$ (d) ∞

9. Residue of $\cot z$ at $z = 0$ is

(a) 0 (b) 1

(c) 2 (d) 2π

10. If Γ lies in a disk which does not contain the origin then $n(\Gamma, 0)$ is

(a) 1 (b) 0

(c) ∞ (d) $2\pi i$

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Obtain the cauchy – Riemann differential equations of an analytic function.

Or

(b) Show that the analytic function cannot have a constant absolute value without reducing to a constant.

12. (a) Find the fixed points of the linear transformation $w = \frac{3z-4}{z-1}$.

Or

(b) Compute $\int_{|z|=2} \frac{dz}{z^2-1}$ for the positive sense of the circle.

13. (a) State and prove the integral formula.

Or

(b) State and prove the fundamental theorem of algebra.

14. (a) State and prove Weierstrassian theorem on the behaviour of a function in the Neighborhood of an essential singularity.

Or

(b) State and prove the maximum principle.

15. (a) State and prove the argument principle.

Or

(b) Find the poles and residues of the functions $\frac{1}{z^2+5z+6}$.

SECTION C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) (i) If all zeros of a polynomial $p(z)$ lie in a half plane, prove that all zeros of the derivative $p'(z)$ lie in the same half plane.

(ii) Prove that $\sum_{0}^{\infty} a_n z^n$ and $\sum_{1}^{\infty} n a_n z^{n-1}$ have the same radius of convergence.

Or

(b) State and prove Abel's limit theorem.

17. (a) If $T_1 z = \frac{z+2}{z+3}$, $T_2 z = \frac{z}{z+1}$, find $T_1 T_2 z$, $T_2 T_1 z$ and $T_1^{-1} T_2 z$.

Or

(b) Obtain a necessary and sufficient condition under which a line integral depends only on the end points.

18. (a) State and prove Cauchy's theorem for a rectangle.

Or

(b) Compute $\int_{|z|=2} \frac{dz}{z^2+1}$ and $\int_{|z|=1} e^z z^{-n} dz$.

19. (a) State and prove Taylor's theorem.

Or

(b) State and prove the lemma of schwarz.

20. (a) Compute $\int_0^{\pi/2} \frac{dx}{a+\sin^2 x}, |a|>1$.

Or

(b) Compute $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9}\,dx$.

(7 pages) **Reg. No. : ....**

Code No. : 7134 **Sub. Code : PMAM 42**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Fourth Semester

Mathematics — Core

COMPLEX ANALYSIS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $u = 2x - x^3 + 3xy^2$ then Δu is

   (a) $12\,x$ (b) 0

   (c) $6x - 6$ (d) $6xy + 6x$.

2. If $P(z) = a_3 (z - \alpha_1)(z - \alpha_2)(z - \alpha_3)$ then $\dfrac{P'(z)}{P(z)}$ is

   (a) $\dfrac{1}{z - \alpha_1}$ (b) $\dfrac{1}{z - \alpha_2}$

   (c) $\dfrac{1}{z - \alpha_3}$ (d) a_3.

3. The complex form of the Cauchy–Riemann equation is

(a) $\frac{\partial f}{\partial x} = i\frac{\partial f}{\partial y}$

(b) $\frac{\partial f}{\partial x} - i\frac{\partial f}{\partial y} = 0$

(c) $\frac{\partial f}{\partial x} = -i\frac{\partial f}{\partial y}$

(d) $\frac{\partial f}{\partial y} = -i\frac{\partial f}{\partial x}$.

4. The points z and z^{α} are symmetric w.r.t. the circle C through z_1, z_2, z_3 if and only it

(a) $\left(z^*, z_1, z_2, z_3\right) = \left(z, z_1, z_2, z_3\right)$

(b) $\left(z^*, z_1, z_2, z_3\right) = -\left(z, z_1, z_2, z_3\right)$

(c) $\left(z, z_1, z_2, z_3\right) = z + z^*$

(d) $\overline{\left(z, z_1, z_2, z_3\right)} = \left(z^*, z_1, z_2, z_3\right)$.

5. $\int_{|z-2|=5} \frac{dz}{z-3}$ is

(a) 1

(b) 0

(c) $2\pi i$

(d) $-2\pi i$.

6. The index of the point a w.r.t. the cure γ is

(a) $\int_{\gamma} \frac{dz}{z-a}$

(b) $2\pi i \int_{\gamma} \frac{dz}{z-a}$

(c) $\frac{1}{2\pi i}\int_{\gamma} \frac{dz}{z-a}$

(d) $\int_{\gamma} (z-a)\, dz$.

7. "A function which is analytic and bounded in the whole plane must reduce to a constant" — This result is known as

(a) Morera's theorem

(b) Liouville's theorem

(c) Fundamental theorem of algebra

(d) Cauchy's theorem.

8. $\int_{|z|=1} e^z \cdot z^{-n} \, dz$ is

(a) 0 (b) $2\pi i$

(c) $\frac{2\pi i}{(n-1)!}$ (d) $\frac{2\pi i}{n!}$.

9. The residue of $\frac{e^z}{(z-a)^2}$ at $z = a$ is

(a) e^a (b) e^{2a}

(c) e^{-2a} (d) e^{-a}.

10. If f has a pole of order h then

(a) f_1/f has the residue $-h$

(b) f_1/f has the residue h

(c) f/f_1 has the residue h

(d) f_1/f has the residue $\frac{-h}{h}$.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions choosing either (a) or (b).

11. (a) If all zeros of a polynomial $P(z)$ lie in a half plane, prove that all zeros of the derivative $P'(z)$ lie in the same half plane.

Or

(b) Verify Cauchy-Riemann's equations for the function z^3.

12. (a) Show that any linear transformation which transforms the real axis into itself can be written with real coefficients.

Or

(b) Find the linear transformation which carries $0, i, -i$ into $1, -1, 0$.

13. (a) Obtain Cauchy's integral formula.

Or

(b) Compute $\int_{|z|=2} \frac{dz}{z^2+1}$.

14. (a) State and prove Liouville's theorem.

Or

(b) State and prove the fundamental theorem of algebra.

15. (a) State and prove the residue theorem.

Or

(b) State and prove the Rouche's theorem.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Define an analytic function with an example. Prove that the functions $f(z)$ and $\overline{f(\overline{z})}$ are simultaneously analytic.

Or

(b) (i) Show that $\sum_{0}^{\infty} a_n z^n$ and $\sum_{1}^{\alpha} n a_n z^{n-1}$ have the same radius of convergence.

(ii) If $\sum_{0}^{\alpha} a_n$ converges, prove that $f(z) = \sum_{u}^{\alpha} a_n z^n \to f(1)$ as $z \to 1$ in such a way that $|1 - z|/(1 - |z|)$ remains bounded.

17. (a) Prove that the line integral $\int_{\gamma} p\,dx + q\,dy$ defined in Ω depends only on the end points of γ if and only if there exists a function $U(x, y)$ in Ω with the partial derivatives $\frac{\partial U}{\partial x} = p$, $\frac{\partial U}{\partial y} = q$.

Or

(b) (i) Define the cross ratio (z_1, z_2, z_3, z_4). Show that the cross ratio is invariant under linear transformation.

(ii) Prove that a linear transformation carries circles into circles.

18. (a) State and prove Cauchy's theorem for a rectangle.

Or

(b) Show that the value of the integral $\int_{\gamma} \frac{dz}{z-a}$ is a multiple of $2\pi i$ and hence show that the winding number $n(\gamma, a)$ is an integer. Also show that $n(-\gamma, a) = -n(\gamma, a)$.

19. (a) State and prove Taylor's theorem.

Or

(b) State and prove Weierstrass theorem for essential singularity of an analytic function.

20. (a) State and prove the argument principle.

Or

(b) Evaluate $\int_{-\alpha}^{\alpha} \frac{x^2 - x + 2}{x^4 + 10x^2 + q} \, dx$.

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(7 pages) **Reg. No. :** ....

Code No. : 7135 **Sub. Code : PMAM 43**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Fourth Semester

Mathematics – IV

ADVANCED ALGEBRA – II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The number e is

   (a) rational (b) algebraic

   (c) trancendental (d) a unit

2. What is the degree of $\sqrt{2}\sqrt{3}$ over Q?

   (a) 1 (b) 2

   (c) 3 (d) 4

3. τ^* is an isomorphism of $F[x]$ onto $F'[t]$ with the property that, for all $\alpha \in F$, $\alpha\tau^* =$

(a) α (b) 0

(c) α' (d) t

4. If $f'(x) = 0$ where $f(x) \in F[x]$ and f is of characteristic 3 then for some polynomial $g(x) \in F[x]$,

(a) $g'(x) = 0$ (b) $f(x^3) = g(x)$

(c) $f(x) = g(x)$ (d) $f(x) = g(x^3)$

5. If $F(x_1, x_2, \ldots x_n)$ is the field of rational functions in $x_1, x_2 \ldots x_n$ over F and S is the field of symmetric rational functions then $[F(x_1, x_2 \ldots x_n) : S] =$

(a) S_n (b) n

(c) $n!$ (d) $G(F(x_1, x_2, \ldots x_n), S)$

6. If F is the field of rational numbers and $K = F(\sqrt[3]{2})$ then $0(G(K, F))$ is

(a) 1 (b) 2

(c) 3 (d) 4

7. If F is a field with 9 elements, $F \subset K$ where K is a finite field such that $[K:F]=2$ then K has ———————— elements.

(a) 7 (b) 18

(c) 512 (d) 81

8. The cyclotomic polynomial $P_6(x)=$

(a) x^2+x-1 (b) $x^4-x^3-x^2+1$

(c) x^2-x+1 (d) x^6-x^3+1

9. The irreducible polynomials over the field of real numbers are of degree

(a) 1 (b) 2

(c) either 1 or 2 (d) neither 1 nor 2

10. If $x \in H$, the Hurwitz ring of integral quaternions $x \neq 0$ then $N(x)$ is

(a) x^* (b) 0

(c) a positive integer (d) can't say

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If L is an algebraic extension of K and if K is an algebraic extension of F, show that L is an algebraic extension of F.

Or

(b) If $V = (g(x))$ is the ideal generated by the polynomial $g(x)$ of degree n in $F[x]$, prove that $\frac{F[x]}{V}$ is an n–dimensional vector space over F.

12. (a) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.

Or

(b) Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a nontrivial common factor.

13. (a) Define the fixed field of a group G of automorphsims of K and show that it is a sub field of K.

Or

(b) If K is a field and if $\sigma_1, \sigma_2, \ldots \sigma_n$ are distinct automorphisms of K, show that it is impossible to find elements $a_1, a_2 \ldots a_n$ not all O, in K such that $a_1\sigma_1(u) + a_2\sigma_2(u) + \ldots + a_n\sigma_n(u) = 0$ for all $u \in K$.

14. (a) Show that for every prime number p and every positive integer m there exists a field having p^m elements.

Or

(b) If F is a finite field and $\alpha \neq 0$, $\beta \neq 0$ are two elements of F, show that there exist elements a and b in F such that $1 + \alpha a^2 + \beta b^2 = 0$.

15. (a) Show that the adjoint in the division ring Q of real quaternions satisfies the following :

(i) $x^{**} = x$

(ii) $(\delta x + \gamma y)^* = \delta x^* + \gamma y^*$

(iii) $(xy)^* = y^* x^*$ for all x, y in Q and all real δ and γ.

Or

(b) Define the norm $N(x)$ in Q and show that, for all x, y in Q $N(xy) = N(x)N(y)$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If L is a finite extension of K and if K is a finite extension of F, show that $[L:F]=[L:K][K:F]$. Draw your inference when $[L:F]$ is a prime number.

Or

(b) If $a \in K$ is algebraic of degree n over F, prove that $[F(a):F]=n$.

17. (a) If $p(x)$ is irreducible in $F[x]$ and if V is a root of $p(x)$ then, show that $F(V)$ is isomorphic to $F'(W)$ where W is a root of $p'(t)$, by an isomorphism σ such that (i) $v\sigma = w$ and (ii) $\alpha\sigma = \alpha'$ for every α in F.

Or

(b) If F is of characteristics O and if a, b are algebraic over F, prove that there exists an element C in $F(a, b)$ such that $F(a, b) = F(c)$.

18. (a) Prove that $[K:F] = O(G(K, F))$, where K is a normal extension of F.

Or

(b) Given $F(x_1, x_2 \ldots x_n)$ is to field of rational functions, S is the field of symmetric rational functions $a_1, a_2, \ldots a_n$. Prove that (i) $S = F(a_1, a_2, \ldots a_n)$ and (ii) $F(x_1, x_2 \ldots x_n)$ is the splitting field over S of the polynomial $t^n - a_1 t^{n-1} + a_2 t^{n-2} + \ldots + (-1)^n a_n$.

19. (a) Given G is a finite abelian group with the property that $x^n = e$ is satisfied by at most n elements of G, for every integer n. Show that G is a cyclic group. Deduce that the multiplicative group of non zero elements of a finite field is cyclic.

Or

(b) State and prove Wedderburn's theorem on finite division kings.

20. (a) State and prove Frobenius theorem.

Or

(b) State and prove Lagrange's four-square theorem.